*** مکانیک کوانتومی ۲ :

* ضرایب کلبش ـ گوردن :

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \longrightarrow J_2, m_2 (J_2^2, J_{2z})$$

$\vec{J} \downarrow J, m$ (J^2, J_z) ; $\vec{J}_1 \downarrow J_1, m_1$ (J_1^2, J_{1z})

(J_2 و J_1 داده شده هستند)

دو تا پایه داریم:

دکارتی $\longrightarrow$ قدیم : $|J_1, J_2, m_1, m_2\rangle$ $\;\llcorner\!\rightarrow = |m_1, m_2\rangle$

قطبی $\longrightarrow$ جدید : $|J, m\rangle$

$$\underbrace{|Jm\rangle}_{\text{پایه جدید}} = \sum_{m_1, m_2} \underbrace{\boxed{|J_1, J_2, m_1, m_2\rangle}}_{\text{پایه قدیم}} \underbrace{\boxed{\langle J_1, J_2, m_1, m_2 | Jm\rangle}}_{\text{ضرایب کلبش ـ گوردن}}$$

$$\{J_1^2, J_2^2, J_{1z}, J_{2z}\} \qquad \sum_{m_1, m_2} |J_1, J_2, m_1, m_2\rangle \langle J_1, J_2, m_1, m_2| = 1$$

$$\{J_1^2, J_2^2, J^2, J_z\} \qquad \sum_{J, m} |Jm\rangle \langle Jm| = 1$$

* رابطه بهنجارش :

$$\sum_{m_1, m_2} \left(\langle J_1 J_2, m_1 m_2 | Jm\rangle \right)^2 = 1$$

$$\sum_{J, m} \left(\langle J_1, J_2, m_1 m_2 | Jm\rangle \right)^2 = 1$$

$$J_z = J_{1z} + J_{2z} \longrightarrow m = m_1 + m_2$$

$J_z \downarrow J, m$; $J_{1z} \downarrow J_1, m_1$ $(m_1 \in (-J_1 + J_1))$; J_{2z}: $\llcorner\!\rightarrow J_2, m_2 (m_2 \in (-J_2, +J_2))$

$\downarrow$ $|m| \leqslant J$

$$m_{1max} = J_1 \qquad m_{2max} = J_2$$

$$m_{max} = J_1 + J_2$$

$$J_{max} = J_1 + J_2$$

* نکته : می‌خواهیم J_{min} را طوری بدست بیاوریم که این رابطه :

$$\sum_{J=J_{min}}^{J_{max}} (2J+1) = (2J_1+1)(2J_2+1)$$

$$(2J_{min}+1)+(2(J_{min}+1)+1)+(2(J_{min}+2)+1)+\dots+(2J_{max}+1)=$$

$$=(2J_{min}+1)+(2J_{min}+3)+(2J_{min}+5)+\dots+(2(J_1+J_2)+1)=$$

$$=(J_{max}-J_{min}+1)(J_{min}+J_{max}+1)=(2J_1+1)(2J_2+1)$$

$\Rightarrow J_{min}=|J_1-J_2| \longrightarrow$ حل مطابق درس ۲

$M_{min}=-(J_1+J_2)$ $\qquad |J_1+J_2| \leqslant J \leqslant J_1+J_2$

$J_1=1 \qquad m_1=-1,0,+1$

$J_2=\frac{1}{2} \qquad m_2=-\frac{1}{2},\frac{1}{2}$

$J=\frac{1}{2},\frac{3}{2}$

$\frac{1}{2} \to -\frac{1}{2},+\frac{1}{2}$

$\frac{3}{2} \to -\frac{3}{2},-\frac{1}{2},+\frac{1}{2},+\frac{3}{2}$

$$\{|Jm\rangle\}=\{|+\tfrac{1}{2},-\tfrac{1}{2}\rangle, |+\tfrac{1}{2},\tfrac{1}{2}\rangle, |\tfrac{3}{2},-\tfrac{3}{2}\rangle, |\tfrac{3}{2},-\tfrac{1}{2}\rangle, |\tfrac{3}{2},\tfrac{1}{2}\rangle, |\tfrac{3}{2},\tfrac{3}{2}\rangle\}$$

$$\{|-1,-\tfrac{1}{2}\rangle, |-1,+\tfrac{1}{2}\rangle, |0,-\tfrac{1}{2}\rangle, |0,+\tfrac{1}{2}\rangle, |1,-\tfrac{1}{2}\rangle, |1,+\tfrac{1}{2}\rangle\}$$

$\langle J_1,J_2;m_1,m_2|Jm\rangle$

$m_1=J_1 \qquad J=J_1+J_2$

$m_2=J_2 \qquad m=J_1+J_2$

$\langle -1,-\frac{1}{2}|\frac{1}{2},-\frac{1}{2}\rangle=0$

$\langle -1,-\frac{1}{2}|\frac{1}{2},\frac{1}{2}\rangle=0$

$\langle -1,-\frac{1}{2}|\frac{3}{2},-\frac{3}{2}\rangle$

$\langle -1,-\frac{1}{2}|\frac{3}{2},-\frac{1}{2}\rangle=0$

$\langle -1,-\frac{1}{2}|\frac{3}{2},\frac{1}{2}\rangle=0$

$\langle -1,-\frac{1}{2}|\frac{3}{2},\frac{3}{2}\rangle=0$

$$|\overbrace{J_1+J_2}^{J},\overbrace{J_1+J_2}^{m}\rangle=\sum_{m_1,m_2}|J_1,J_2,m_1,m_2\rangle\boxed{\langle J_1,J_2,m_1,m_2|J_1+J_2,J_1+J_2\rangle}$$

$\boxed{m_1+m_2=m}$

α

$$=|J_1,J_2,J_1,J_2\rangle\boxed{\langle J_1,J_2,J_1,J_2|J_1+J_2,J_1+J_2\rangle}$$

$$\langle j_1+j_2, j_1+j_2 | j_1, j_2, j_1+j_2\rangle = \alpha^2 \underbrace{\langle j_1, j_2, j_1, j_2 | j_1, j_2, j_1, j_2\rangle}_{1} = \alpha^2$$

$$\Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$$

$$\langle j_1, j_2, j_1, j_2 | j_1+j_2, j_1+j_2\rangle = 1 \Rightarrow$$ بزرگترین ضریب CG

$$\langle j_1, j_2, -j_1, -j_2 | j_1+j_2, -j_1, -j_2\rangle = 1 \Rightarrow$$ کوچکترین ضریب CG

$$J_\pm |j m\rangle = \hbar\sqrt{(j\mp m)(j\pm m+1)}\,|j, m\pm 1\rangle \Rightarrow$$ بلدباشیم

$$\langle j_1, j_2, m_1, m_2 | J_\pm | j, m\rangle = \hbar\sqrt{(j\mp m)(j\pm m+1)}$$ ادامه

$$\langle j_1, j_2, m_1, m_2 | j, m\pm 1\rangle$$

$$= \hbar\sqrt{(j_1\pm m_1)(j_1\mp m_1+1)}\,\langle j_1, j_2, m_1\mp 1, m_2 | j m\rangle + \hbar\sqrt{(j_2\pm m_2)(j_2\mp m_2+1)}$$

$$\langle j_1, j_2, m_1, m_2\mp 1 | j m\rangle$$

* رابطه مهم :

$$\sqrt{(j\mp m)(j\pm m+1)}\,\langle j_1, j_2, m_1, m_2 | j, m\pm 1\rangle = \sqrt{(j_1\pm m_1)(j_1\mp m_1+1)}$$

$$\langle j_1, j_2, m_1\mp 1, m_2 | j m\rangle + \sqrt{(j_2\pm m_2)(j_2\mp m_2+1)} =$$

$$\langle j_1, j_2, m_1, m_2\mp 1 | j m\rangle$$

در این رابطه به جای m عبارت $(m\mp 1)$ می‌گذاریم :

در نتیجه داریم $\Leftarrow$

$$\sqrt{(j\mp m+1)(j\pm m)}\quad \langle j_1, j_2, m_1, m_2 | j, m\rangle = \sqrt{(j_1\pm m_1)(j_1\mp m_1+1)}$$

$$\langle j_1, j_2, m_1\mp 1, m_2 | j, m\mp 1\rangle +$$

$$\sqrt{(j_2\pm m_2)(j_2\mp m_2+1)}\,\langle j_1, j_2, m_1, m_2\mp 1 | j, m\mp 1\rangle$$

مجموع دو تا اسپین $\frac{1}{2}$

$$\begin{cases} J_1 = \frac{1}{2} \longrightarrow m_1 = -\frac{1}{2}, +\frac{1}{2} = \downarrow, \uparrow \\ J_2 = \frac{1}{2} \longrightarrow m_2 = -\frac{1}{2}, \frac{1}{2} = \downarrow, \uparrow \end{cases}$$

$$\{|\downarrow,\downarrow\rangle, \langle\downarrow,\uparrow\rangle, |\uparrow,\downarrow\rangle, |\uparrow,\uparrow\rangle\}$$ پایه قدیم

$$0 \leq J \leq 1$$

$$\begin{cases} J=0 \longrightarrow m=0 \quad \text{singlet} \quad |0,0\rangle \\ J=1 \longrightarrow m=-1,0,+1 \quad \text{triplet} \quad \{|1,-1\rangle, |1,0\rangle, |1,+1\rangle\} \end{cases}$$ پایه جدید

$$|J,m\rangle = \sum_{m_1,m_2} |m_1,m_2\rangle\langle m_1,m_2|Jm\rangle$$

$$|00\rangle = \overset{a}{\boxed{\langle\uparrow\downarrow|00\rangle}}|\uparrow\downarrow\rangle + \overset{b}{\boxed{\langle\downarrow\uparrow|00\rangle}}|\downarrow\uparrow\rangle$$

$$|1,1\rangle = \overset{=1}{\boxed{\langle\uparrow\uparrow|11\rangle}}|\uparrow\uparrow\rangle$$

$$|1,0\rangle = \overset{c}{\boxed{\langle\uparrow\downarrow|10\rangle}}|\uparrow\downarrow\rangle + \overset{d}{\boxed{\langle\downarrow\uparrow|10\rangle}}|\downarrow\uparrow\rangle$$

$$|1,-1\rangle = \boxed{\langle\downarrow\downarrow|1,-1\rangle}|\downarrow\downarrow\rangle$$

$$|00\rangle = a|\uparrow\downarrow\rangle + b|\downarrow\uparrow\rangle$$

$$|11\rangle = |\uparrow\uparrow\rangle \longrightarrow J_-|11\rangle = J_-|11\rangle$$

$$|1,0\rangle = c|\uparrow\downarrow\rangle + d|\downarrow\uparrow\rangle$$

$$|1,-1\rangle = |\downarrow\downarrow\rangle$$

$$J_-|Jm\rangle = \hbar\sqrt{(J+m)(J-m+1)}\,|J,m-1\rangle \qquad \langle 00|00\rangle = 1$$

$$\hbar\sqrt{2\times 1}\,|10\rangle = \hbar\sqrt{1\times 1}\,|\downarrow\uparrow\rangle + \hbar\sqrt{1\times 1}\,|\uparrow\downarrow\rangle$$

$$\hbar\sqrt{2}\,|10\rangle = \hbar|\downarrow\uparrow\rangle + \hbar|\uparrow\downarrow\rangle$$

$$\Rightarrow |10\rangle = \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle + \frac{1}{\sqrt{2}}\uparrow\downarrow\rangle \Rightarrow c = \frac{1}{\sqrt{2}}, d = \frac{1}{\sqrt{2}}, c=d$$

$$\langle 00|00\rangle = 1 = (a\langle\uparrow\downarrow + b\langle\downarrow\uparrow|)(a|\uparrow\downarrow\rangle + b|\downarrow\uparrow\rangle) = a$$

$$a^2\underbrace{\langle\uparrow\downarrow|\uparrow\downarrow\rangle}_{1} + ab\underbrace{\langle\uparrow\downarrow|\downarrow\uparrow\rangle}_{0} + ba\underbrace{\langle\uparrow\downarrow|\downarrow\uparrow\rangle}_{0}\, b^2\underbrace{\langle\downarrow\uparrow|\downarrow\uparrow\rangle}_{1} \qquad 1 = a^2 + b^2$$

$$x_{00}|11\rangle = 0 = \frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}} \quad \boxed{a = -b} \Rightarrow b = -\frac{1}{\sqrt{2}}$$