*** مکانیک کوانتوم ۲ :

- نظریه اختلال مستقل از زمان غیر تبهگن :

$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

$$H = \underbrace{H_0}_{\text{بلدیم}} + \underbrace{H_p}_{\text{خیلی کوچک }(\lambda w)}$$

$$\text{غیر تبهگن} \rightarrow H_0|\phi_n\rangle = E_n^0|\phi_n\rangle$$

$$1 = \sum_n |\phi_n\rangle\langle\phi_n|$$

$$\langle\phi_n|\psi_n^1\rangle = 0$$

$$E_n = E_n^0 + \langle\phi_n|H_p|\phi_n\rangle$$

$$|\psi_n\rangle = |\phi_n\rangle + \sum_{m\neq n} \frac{\langle\phi_m|H_p|\phi_n\rangle}{E_n^0 - E_n^0}|\phi_m\rangle$$

غیر تبهگن

* مرتبه دوم اختلال :

$$\langle\phi_n|w|\psi_n^1\rangle = E_n^2$$

$$\langle\phi_n| \times$$

$$H_0|\psi_n^2\rangle + w|\psi_n^1\rangle = E_n^0|\psi_n^2\rangle + E_n^1|\psi_n^1\rangle + E_n^2|\phi_n\rangle$$

$$E_n^2 = \langle\phi_n|w|\sum_{m\neq n} \frac{\langle\phi_m|w|\phi_n\rangle}{E_n^0 - E_m^0}|\phi_m\rangle \neq \sum_{m\neq n} \frac{\langle\phi_m|w|\phi_n\rangle}{E_n^0 - E_n^0}\langle\phi_n|w|\phi_m\rangle$$

$$= \sum_{m\neq 0} \frac{\langle\phi_m|w|\phi_n\rangle\langle\phi_n|w|\phi_m\rangle}{E_n^0 - E_n^0} = \sum_{m\neq n} \frac{|\langle\phi_m|w|\phi_n\rangle|^2}{E_n^0 - E_n^0}$$

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* یک بار q با جرم m در یک میدان الکتریکی ضعیف ε قرار دارد و شروع به نوسان می کند :

- جمله برهم کنش بار و میدان الکتریکی : $\vec{\varepsilon}\cdot\vec{p}$

$$\langle\phi_n|H_p|\phi_n\rangle = \int d^3r\, \phi_n^*(\vec{r})\, H_p(\vec{r})\, \phi_n(-\vec{r})$$

$$H = \underbrace{\frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2}_{} + q\varepsilon x$$

اختلالی H_p

$$N = a^+ a \qquad N|n\rangle = n|n\rangle$$

$$\begin{cases} a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases}$$

$$E_n^0 = \hbar\omega\left(n + \frac{1}{2}\right)^*$$

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$$\underbrace{\frac{1}{2} m\omega^2 \left(x^2 + \frac{2q\varepsilon}{m\omega^2}x + \frac{q^2\varepsilon^2}{m^2\omega^4}\right)}_{\left(x+\frac{q\varepsilon}{m\omega^2}\right)^2} - \underbrace{\frac{1}{2}m\omega^2\left(\frac{q^2\varepsilon^2}{m^2\omega^4}\right)}_{-\frac{q^2\varepsilon^2}{2m\omega^2}}$$

$H_p = q\varepsilon x$

$$E_n^1 = \langle n|q\varepsilon x|n\rangle = q\varepsilon\langle n|x|n\rangle = q\varepsilon\sqrt{\frac{\hbar}{2m\omega}}\langle n|a^\dagger + a|n\rangle = 0$$

$$= q\varepsilon\sqrt{\frac{\hbar}{2m\omega}}\left(\underbrace{\langle n|a^\dagger|n\rangle}_{=0} + \underbrace{\langle n|a|n\rangle}_{=0}\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a)$$

$\sqrt{n+1}\,|n+1|$

$\sqrt{n+1}\underbrace{\langle n|n+1\rangle}$

E_n^2

$\langle n|m\rangle = \delta_{nm}$

$$E_n^2 = \sum_{m\neq n}\frac{|\langle\phi_m|w|\phi_n\rangle|^2}{} = q^2\varepsilon^2\sum_{m\neq n}\frac{|\langle m|x|n\rangle|^2}{E_n^0-E_n^0}$$

$$\langle m|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\langle m|a+a^\dagger|n\rangle = q^2\varepsilon^2\left(\frac{|\langle n+1|x|n\rangle|^2}{E_n^0-E_{n+1}^0} + \frac{|\langle n-1|x|n\rangle|^2}{E_2^0-E_{n-1}^0}\right) =$$

$m \neq n$

$$\Rightarrow -\frac{q^2\varepsilon^2}{2m\omega^2}$$

$$\langle n+1|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\langle n+1|a^\dagger + a|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\sqrt{n+1}$$

$$|\psi_n^1\rangle = \sum_{m\neq n}\frac{\langle\phi_m|w|\phi_n\rangle}{E_n^0-E_m^0}|\phi_m\rangle = q\varepsilon\left(\frac{\langle n+1|x|n\rangle}{E_n^0-E_{n+1}^0}|n+1\rangle + \frac{\langle n-1|x|n\rangle}{E_n^0-E_{n-1}^0}|n-1\rangle\right) =$$

$$= \frac{q\varepsilon}{\hbar\omega}\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\,|n-1\rangle - \sqrt{n+1}\,|n+1\rangle\right)$$

*** اثر اشتارک (stark effect):**

- این اثر میدان الکتریکی روی اتم است.

$$\vec{\mathcal{E}} = \varepsilon\hat{k}$$

$$H_0 = \frac{p^2}{2\mu} - \frac{e^2}{r} \qquad \psi_{n\ell m}(r|\theta|\varphi) = R_{n\ell}(r)\,Y_{\ell m}(\theta,\varphi)$$

(e در حالت های برانگیخته از اتم هیدروژن تبهگن است.)

$$H_p = q\vec{\mathcal{E}}\cdot\vec{r} = -e\varepsilon z = -e\varepsilon r\cos\theta$$

$$E_{100} = E^0_{100} + e\varepsilon\langle 100|z|100\rangle = E^0_{100} + e\varepsilon\int \psi^*_{100}(r)\,z\,\psi_{100}(r)\,d^3r =$$

$$= E^0_{100} + e\varepsilon\int|\psi_{100}|^2 z\,d^3r = E^0_{100}$$

$$E^2_n = e^2\varepsilon^2\sum_{n\ell m\neq 100}\frac{|\langle n\ell m|z|100\rangle|^2}{E^0_{100}-E^0_{n\ell m}} \geqslant e^2\varepsilon^2\sum\frac{|\langle n\ell m|z|100\rangle|^2}{E^0_{100}-E^0_{200}}$$

$$E^0_{100}-E^0_{n\ell m} \leqslant E^0_{100}-E^0_{200} = \frac{e^2}{2a_0}\left(-1-\left(-\frac{1}{4}\right)\right) = \frac{3e^2}{8a_0}$$

$$= \frac{e^2\varepsilon^2}{\frac{-3e^2}{8a_0}}\sum_{n\ell m\neq 0}\langle 100|z|n\ell m\rangle\langle n\ell m|z|100\rangle = \frac{-8\varepsilon^2 a_0}{3}\langle 100|z^2|100\rangle =$$

$$= \frac{-8\varepsilon^2 a_0}{3}\int R^2_{10}(r)\,r^4\,dr \times \int \underbrace{Y^2_{00}(\theta,\varphi)}_{\left(\sqrt{\frac{1}{4\pi}}\right)^2}\cos^2\theta\,d\Omega = \frac{-8\varepsilon^2 a_0^3}{3}$$

$$= E^2_n \geqslant \frac{-8\varepsilon^2 a_0^3}{3}$$

نظریه اختلال مستقل از زمان تبهگن:

$$H|\psi_n\rangle = E_n|\psi_n\rangle \qquad \langle\psi_n|\psi_n\rangle = 1$$

$$H = H_0 + H_p$$

هامیلتونی غیراختلالی تبهگن است.

$$\langle\psi_{nf}|\phi_{nf}\rangle = 1$$

$$H_0|\phi_{nf}\rangle = E_n^0|\phi_{nf}\rangle \qquad \langle\phi_{nf}|\psi_n^*\rangle = 0$$

$$|\psi_n^0\rangle = \sum_f \alpha_f|\phi_{nf}\rangle \;\Rightarrow\; \sum \alpha^2_f = 1 \qquad f = 1, \dots, N$$

درجه تبهگنی

$$|\psi_n\rangle = \sum_f \alpha_f|\phi_{nf}\rangle + \lambda|\psi_n^1\rangle + \lambda^2|\psi_n^2\rangle + \dots$$

$$E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \dots$$

$$(H_0 + \lambda W)\left(\sum_f \alpha_f|\phi_{nf}\rangle + \lambda|\psi_n^1\rangle + \lambda^2|\psi_n^2\rangle + \dots\right) =$$

$$(E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \dots)$$

$$E_n^0\cancel{\langle\phi_{n_1}|\psi_n^1\rangle} \to 0 \qquad \times\left(\sum_f \alpha_f|\phi_{nf}\rangle + \lambda|\psi_n^1\rangle + \lambda^2|\psi_n^2\rangle \dots\right)$$

$$H_0|\psi_n^1\rangle + \sum_f \alpha_f W|\phi_{nf}\rangle = E_n^0|\phi_n^1\rangle + \sum_f \alpha_f E_n^1|\phi_{nf}\rangle$$

$$\langle\phi_{ni}|H_0|\psi_n^1\rangle + \sum_f \alpha_f \overbrace{\langle\phi_{ni}|W|\phi_{nf}\rangle}^{W_{if}} = E_n^0\langle\phi_{ni}|\psi_n^1\rangle + \sum_f \alpha_f E_n^1 \overbrace{\langle\phi_{ni}|\phi_{nf}\rangle}^{\delta_{if}}$$

$$\sum_f \alpha_f (W_{if} - E_n^1\delta_{if}) = 0 \Rightarrow \det(W_{if} - E_n^1\delta_{if}) = 0$$

انرژی های اختلال (مرتبه ۱) ویژه مقادیر ماتریس W هستند.

$$\det(A - \lambda I) = 0$$