# Cosmology of Galileons in Mimetic Gravity

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# The mimetic DM theory

- Define an effective metric $g_{\mu\nu}$ as

$$g_{\mu\nu} = -\partial_\alpha\phi\partial_\beta\phi\hat{g}^{\alpha\beta}\hat{g}_{\mu\nu}.$$

- The above metric is invariant under the conformal transformation

$$\hat{g}_{\mu\nu} \to \Omega^2(x)\hat{g}_{\mu\nu}.$$

- Any theory written in terms of $g_{\mu\nu}$ will be conformally invariant (Mukhanov 2013)
- Consider the Einstein-Hilbert action

$$S = \int d^4x\sqrt{-g}R(g_{\mu\nu}) + S_m.$$

# Mimetic DM

- The metric field equation is

$$G_{\mu\nu} - T_{\mu\nu} + (R + T)\partial_\mu\phi\partial_\nu\phi = 0.$$

- The scalar field equation is

$$\nabla_\mu \Big[ (R + T)\partial^\mu\phi \Big] = 0.$$

- From the first equation one can define an effective energy-momentum tensor

$$\bar{T}_{\mu\nu} = -(R + T)\partial_\mu\phi\partial_\nu\phi,$$

# Mimetic DM

- It represents dust with

$$\rho = -(R+T), \quad u_\mu = \partial_\mu \phi, \quad p = 0$$

- One can easily prove that

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1 \Rightarrow u_\mu u^\mu = -1$$

- Also $\nabla_\mu \bar{T}^{\mu\nu} = 0$. The effective energy-momentum tensor is conserved.

# Generalizing Mimetic DM

- The geometric dark matter can also obtained from the action

$$S = \int d^4x\sqrt{-g}\Big[R + \lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + L_m\Big].$$

- The Lagrange multiplier enforces that $\partial_\mu\phi$ is time-like.
- This breaks the Lorentz invariance dynamically; defining a preferred direction.
- There is a shift symmetry on the scalar field: $\phi \to \phi + const.$
- The theory resembles the Horava-Lifshitz gravity.

# Generalizing Mimetic DM

- Dust can not produce a self-accelerated expanding universe.
- We want to generalize the Mimetic action such that:
  - It can also explain the self-accelerated expanding universe.
  - Preserve the shift symmetry on $\phi$.
- One may add some higher derivative self interactions of $\phi$.
- The best candidate is the Galileons.
- Galileons are scalar fields which has higher order self-interactions in the action, but have at most second order time derivatives in the equations of motion.

# The Model

- The action

$$S = \int d^4x\sqrt{-g}\left[\kappa^2 R + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4 + \alpha_5\mathcal{L}_5 + \lambda(\phi_\mu\phi^\mu + 1) + \mathcal{L}_m\right],$$

- where we have

$$\mathcal{L}_3 = (\phi_\alpha\phi^\alpha)\Box\phi,$$

$$\mathcal{L}_4 = (\phi_\alpha\phi^\alpha)\left[2(\Box\phi)^2 - 2\phi_{\mu\nu}\phi^{\mu\nu} - \frac{1}{2}(\phi_\mu\phi^\mu)R\right],$$

$$\mathcal{L}_5 = (\phi_\alpha\phi^\alpha)\left[(\Box\phi)^3 - 3(\phi_{\mu\nu}\phi^{\mu\nu})\Box\phi + 2\phi_\mu{}^\nu\phi_\nu{}^\rho\phi_\rho{}^\mu - 6\phi_\mu\phi^{\mu\nu}G_{\nu\rho}\phi^\rho\right].$$

- $\mathcal{L}_m$ is the matter Lagrangian, and $\phi_\mu \equiv \nabla_\mu\phi$.
- Varying wrt $\lambda$ gives

$$\phi_\mu\phi^\mu = -1,$$

# The Model

- The metric field equations is

$$\kappa^2 G_{\mu\nu} = T_{\mu\nu} + \alpha_3 T^3_{\mu\nu} + \alpha_4 T^4_{\mu\nu} + \alpha_5 T^5_{\mu\nu} - \lambda\phi_\mu\phi_\nu,$$

- where

$$\begin{aligned}
T^3_{\mu\nu} &= -\phi_\mu\phi_\nu\Box\phi,\\
T^4_{\mu\nu} &= -2\phi_{\mu\nu}\Box\phi + 2\phi_{\mu\rho}\phi^\rho{}_\nu + \left((\Box\phi)^2 - \phi_{\rho\sigma}\phi^{\rho\sigma}\right)(g_{\mu\nu} - 2\phi_\mu\phi_\nu)\\
&\quad - \phi_\mu\phi_\nu R + 2\phi^\rho(R_{\rho\mu}\phi_\nu + R_{\rho\nu}\phi_\mu) + \frac{1}{2}G_{\mu\nu} - 2\phi_\rho R^{\rho\sigma}\phi_\sigma g_{\mu\nu} + 2\phi^\rho\phi^\sigma R_{\mu\rho\nu\sigma},\\
T^5_{\mu\nu} &= -3\left((\Box\phi)^2 - \phi_{\sigma\lambda}\phi^{\sigma\lambda}\right)\phi_{\mu\nu} + \Box\phi\left(6\phi_{\mu\sigma}\phi^\sigma{}_\nu - \frac{3}{2}\phi_\mu\phi_\nu R\right) + 3\phi^\sigma\Box\phi(R_{\sigma\mu}\phi_\nu + R_{\sigma\nu}\phi_\mu)\\
&\quad + 3\Box\phi\phi^\sigma\phi^\lambda R_{\mu\sigma\nu\lambda} - 6\phi_{\mu\sigma}\phi^{\sigma\rho}\phi_{\rho\nu} + 3\phi_{\sigma\lambda}R^{\sigma\lambda}\phi_\mu\phi_\nu - 3\phi_\sigma R^{\sigma\lambda}(\phi_{\lambda\mu}\phi_\nu + \phi_{\lambda\nu}\phi_\mu)\\
&\quad - 3\phi^\sigma\phi^{\lambda\rho}(R_{\mu\lambda\sigma\rho}\phi_\nu + R_{\nu\lambda\sigma\rho}\phi_\mu) + 3\phi^\sigma\phi^\lambda(R_{\mu\sigma\lambda\rho}\phi^\rho{}_\nu + R_{\nu\sigma\lambda\rho}\phi^\rho{}_\mu)\\
&\quad + \left((\Box\phi)^3 - 3\Box\phi(\phi_{\rho\sigma}\phi^{\rho\sigma}) + 2\phi_{\rho\sigma}\phi^{\sigma\lambda}\phi_\lambda{}^\rho\right)(g_{\mu\nu} - \phi_\mu\phi_\nu)\\
&\quad + 3\phi_\sigma\phi_\lambda\phi_{\rho\kappa}R^{\sigma\rho\lambda\kappa}g_{\mu\nu} + 3(\phi_\sigma R^{\sigma\lambda}\phi_\lambda)(\phi_{\mu\nu} - \Box\phi g_{\mu\nu}).
\end{aligned}$$

# The Model

- The scalar field equation is

$$\alpha_3\mathcal{E}_3 + \alpha_4\mathcal{E}_4 + \alpha_5\mathcal{E}_5 - 2\nabla_\mu\left(\lambda\phi^\mu\right) = 0,$$

where we have defined

$$\begin{aligned}
\mathcal{E}_3 =& -2(\Box\phi)^2 + 2R_{\mu\nu}\phi^\mu\phi^\nu + 2\phi_{\mu\nu}\phi^{\mu\nu},\\
\mathcal{E}_4 =& -4(\Box\phi)^3 - 8\phi_{\mu\nu}\phi^{\nu\sigma}\phi_\sigma{}^\mu + 12\Box\phi(\phi_{\mu\nu}\phi^{\mu\nu}) - 2(\Box\phi)R\\
&+ 8(\Box\phi)\phi_\mu R^{\mu\nu}\phi_\nu + 4\phi_{\mu\nu}R^{\mu\nu} - 8\phi_\mu\phi_\nu\phi_{\sigma\rho}R^{\mu\rho\nu\sigma},\\
\mathcal{E}_5 =& -2(\Box\phi)^4 + 3(\Box\phi)^2\left(4\phi_{\mu\nu}\phi^{\mu\nu} - R + 2\phi_\mu R^{\mu\nu}\phi_\nu\right) - 16\Box\phi(\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho{}^\mu) + 12(\Box\phi)\phi_{\mu\nu}R^{\mu\nu}\\
&- 12(\Box\phi)\phi_\mu\phi_\nu\phi_{\rho\sigma}R^{\mu\rho\nu\sigma} - 6(\phi_{\mu\nu}\phi^{\mu\nu})(\phi_{\rho\sigma}\phi^{\rho\sigma}) + 12\phi_{\mu\nu}\phi^{\nu\rho}\phi_{\rho\sigma}\phi^{\sigma\mu} + 3(\phi_{\mu\nu}\phi^{\mu\nu})R\\
&- 6(\phi_{\mu\nu}\phi^{\mu\nu})(\phi_\rho R^{\rho\sigma}\phi_\sigma) - 12\phi_{\nu\rho}R^{\rho\sigma}\phi_\sigma{}^\nu - 6\phi_{\nu\rho}\phi_{\sigma\lambda}R^{\nu\sigma\rho\lambda} + 12\phi_\mu\phi_\nu\phi_{\rho\sigma}\phi^\sigma{}_\lambda R^{\mu\rho\nu\lambda}\\
&+ 3(\phi_\nu R^{\nu\rho}\phi_\rho)R - 6\phi_\nu R^{\nu\rho}R_{\rho\sigma}\phi^\sigma - 6\phi_\nu\phi_\rho R_{\sigma\lambda}R^{\nu\sigma\rho\lambda} + 3\phi_\nu\phi_\rho R^\nu{}_{\sigma\kappa\lambda}R^{\rho\sigma\kappa\lambda}.
\end{aligned}$$

# The Model

- One can easily see that

$$\nabla^{\mu}T_{\mu\nu} = -\frac{1}{2}\phi_{\nu}\big[\alpha_3\mathcal{E}_3 + \alpha_4\mathcal{E}_4 + \alpha_5\mathcal{E}_5 - 2\nabla_{\mu}(\lambda\phi^{\mu})\big].$$

- The energy momentum tensor is conserved, after using the scalar field equation.

# Cosmological implications

- Assume

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2).$$

- And a perfect fluid

$$T^{\mu}{}_{\nu} = \mathrm{diag}\big[-\rho(t), p(t), p(t), p(t)\big],$$

- The constraint equation gives

$$\phi = t + c_1,$$

- The shift symmetry in $\phi$ can be used to set $c_1 = 0$ without loosing any generality.

# Cosmological implications

- The field equations are reduced to

$$\frac{3}{2}(15\alpha_4 + 2\kappa^2)H^2 - 21\alpha_5 H^3 - 3\alpha_3 H - \rho + \lambda = 0,$$

$$(3\alpha_4 + 2\kappa^2 - 6\alpha_5 H)\dot{H} + \frac{3}{2}(3\alpha_4 + 2\kappa^2)H^2 - 6\alpha_5 H^3 + p = 0,$$

and

$$6(15\alpha_5 H^2 - 12\alpha_4 H + \alpha_3)\dot{H} + 90\alpha_5 H^4 - 108\alpha_4 H^3 + 18\alpha_3 H^2 - 2(3\lambda H + \dot{\lambda}) = 0.$$

# Cosmological implications

- Assume that $p = \omega\rho$.
- One can solve equations for $\rho$ and $\lambda$ with the result

$$\lambda = 3H(\alpha_3 - 6\alpha_4 H + 5\alpha_5 H^2) + \frac{c_2}{a^3},$$

$$\rho = \frac{3}{2}H^2\left(3\alpha_4 + 2\kappa^2 - 4\alpha_5 H\right) + \frac{c_2}{a^3}.$$

- And the scale factor can be obtained as

$$(2\kappa^2 + 3\alpha_4 - 6\alpha_5 H)\dot{H} + \frac{3}{2}(\omega + 1)\left(2\kappa^2 + 3\alpha_4 - 4\alpha_5 H\right)H^2 + \frac{\omega c_2}{a^3} = 0.$$

# Cosmological implications

- Writing the evolution equation in terms of redshift

$$(1+z)h(z)\left[1+m-nh(z)\right]\frac{dh}{dz} = \frac{3}{2}(\omega+1)\left[1+m-\frac{2}{3}nh(z)\right]h^2(z)+\omega s(1+z)^3.$$

- The energy density

$$r(z)=\frac{3}{2}\left[1+m-\frac{2}{3}nh(z)\right]h^2(z)+s(1+z)^3.$$

- And the Lagrange multiplier

$$\Lambda(z)=3h(z)\left[u-2mh(z)+\frac{5}{6}nh^2(z)\right]+s\,(1+z)^3\,.$$

# Cosmological implications

- We have made the definitions

$$\alpha_3 = 2\kappa^2 u H_0, \quad \alpha_4 = \frac{2\kappa^2}{3} m, \quad \alpha_5 = \frac{\kappa^2}{3}\frac{n}{H_0}, \quad c_2 = 2\kappa^2 H_0^2 s,$$

$$n_1 = \frac{n}{H_0}, \quad \rho(z) = 2\kappa^2 H_0^2 r(z), \quad \lambda = 2\kappa^2 H_0^2 \Lambda, \quad H(z) = H_0 h(z).$$

- The deceleration parameter can be obtain as

$$q(z) = \frac{3}{2}(\omega + 1)\frac{1 + m - (2/3)\, nh(z)}{1 + m - nh(z)} + \frac{\omega s\,(1+z)^3}{h^2(z)\,[1 + m - nh(z)]} - 1.$$

# Dust dominated universe

- The evolution equation reads

$$(1+m-n_1 H)\dot{H}+\frac{3}{2}\left[1+m-\frac{2}{3}n_1 H\right]H^2=0,$$

- Which admits a de Sitter expansion

$$H_0=\frac{3(1+m)}{2n_1}.$$

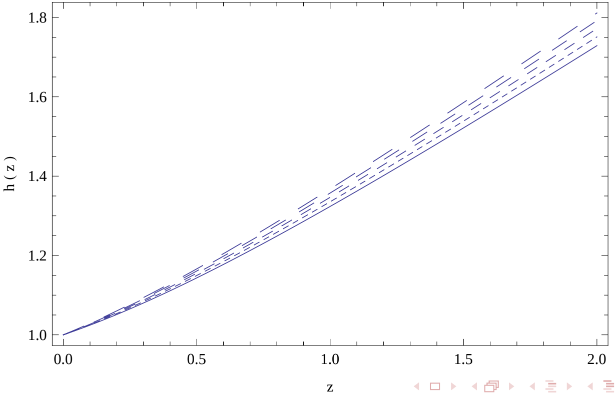
- The general solution is

$$\frac{3}{2}(t-t_0)=\frac{1}{H}+\frac{n_1}{3(1+m)}\ln\frac{H}{3+2m-2n_1 H}.$$

# Dust dominated universe

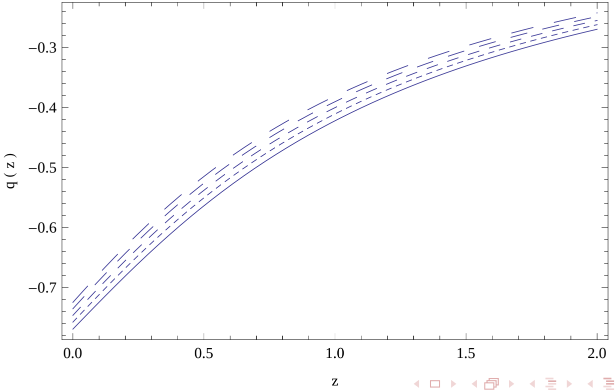
- For $m = 0.002$, $s = 0.20$, and $n = 1.653$ (solid curve), $n = 1.663$ (dotted curve), $n = 1.673$ (short dashed curve), $n = 1.683$ (dashed curve), and $n = 1.693$, respectively.

- The Hubble parameter



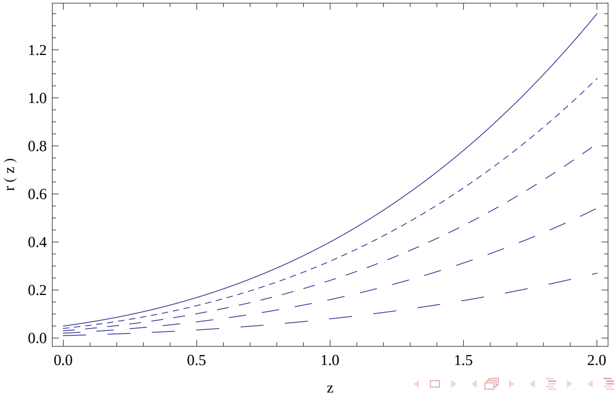
# Dust dominated universe

- The deceleration parameter



# Dust dominated universe

- The energy density



# A Matter-scalar coupling

- Assume that the ordinary matter and the scalar field couple through a term $\alpha_6 \phi T$, with $T$ is the trace of energy-momentum tensor.
- This will add $\alpha_6 T$ to the scalar field equation and

$$\alpha_6 \left( \phi T_{\mu\nu} + \frac{1}{2}(T - 2\mathcal{L}_m)\phi g_{\mu\nu} + 2\phi g^{\alpha\beta} \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} g^{\alpha\beta}} \right)$$

  to the metric equation.
- For dust particles, the last term vanishes.

# A Matter-scalar coupling

- The energy momentum tensor is no longer conserved

$$\nabla^\mu T_{\mu\nu} = \frac{\alpha_6}{1+\alpha_6\phi}\Big[\phi^\mu(g_{\mu\nu}\mathcal{L}_m - T_{\mu\nu}) + \phi\nabla_\nu(\mathcal{L}_m - \frac{1}{2}T) - 2\nabla^\mu(\phi B_{\mu\nu})\Big],$$

with $B_{\mu\nu} = g^{\alpha\beta}\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}$.

- A test particle will move in a non-geodesic path

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^\lambda{}_{\rho\sigma}\frac{dx^\rho}{ds}\frac{dx^\sigma}{ds} = -\frac{h^{\nu\lambda}}{\rho}\frac{\alpha_6}{1+\alpha_6\phi}\Big[\frac{1}{2}\phi\nabla_\nu\rho + \rho\phi_\nu\Big].$$

# Cosmological implications

- Define

$$\alpha_3 = \frac{2\kappa^2}{3} H_0 \eta,\ \alpha_4 = \frac{2\kappa^2}{15}\sigma,\ \alpha_5 = \frac{2\kappa^2}{21 H_0}\theta,\ H = H_0 h,$$
$$\rho = 2\kappa^2 H_0^2 r,\ p = 2\kappa^2 H_0^2 P,\ \lambda = 2\kappa^2 H_0^2 \Lambda,\ \alpha_6 = H_0 \Delta,\ t = \frac{\tau}{H_0},$$

- Field equations become

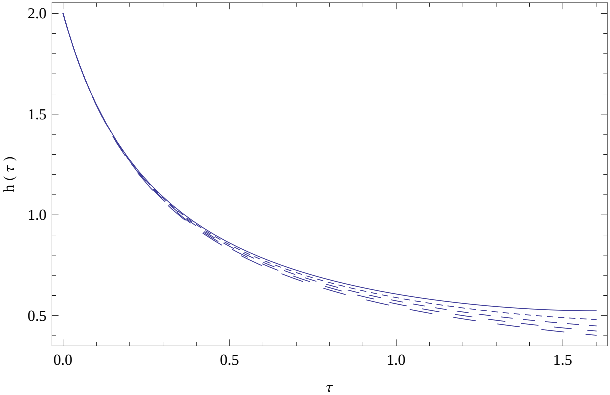
$$\frac{3}{2}(1+\sigma)h^2(\tau) - \theta h^3(\tau) - \eta h(\tau) - r(\tau) + \Lambda(\tau) + \frac{\Delta}{2}\left[3P(\tau) - r(\tau)\right]\tau = 0,$$

$$\left[1 + \frac{\sigma}{5} - \frac{2}{7}\theta h(\tau)\right]\frac{dh(\tau)}{d\tau} + \frac{3}{2}\left(1 + \frac{\sigma}{5}\right)h^2(\tau) - \frac{2}{7}\theta h^3(\tau) + P + \frac{\Delta}{2}\left[5P(\tau) + r(\tau)\right]\tau = 0,$$

$$6\left[\frac{5}{7}\theta h^2(\tau) - \frac{4}{5}\sigma h(\tau) + \frac{\eta}{3}\right]\frac{dh(\tau)}{d\tau} + \frac{30}{7}\theta h^4(\tau) - \frac{36}{5}\sigma h^3(\tau)$$
$$+ 6\eta h^2(\tau) - 2\left[3\Lambda(\tau)h(\tau) + \frac{d\Lambda(\tau)}{d\tau}\right] - \Delta\left[3P(\tau) - r(\tau)\right] = 0.$$

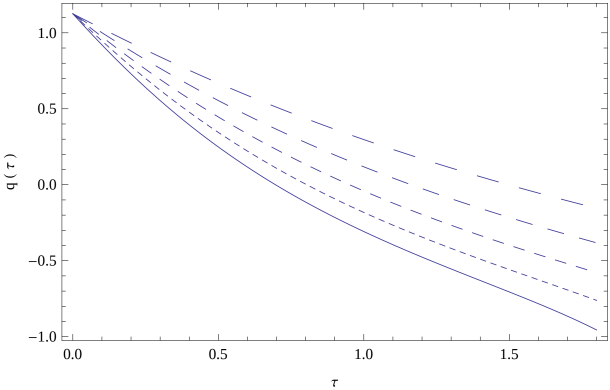
# Dust dominated universe

- For $\Delta = -0.75$ (solid curve), $\Delta = -0.65$, (dotted curve), $\Delta = -0.55$ (short dashed curve), $\Delta = -0.45$ (dashed curve), and $\Delta = -0.35$ (long dashed curve), respectively.
- The Hubble parameter



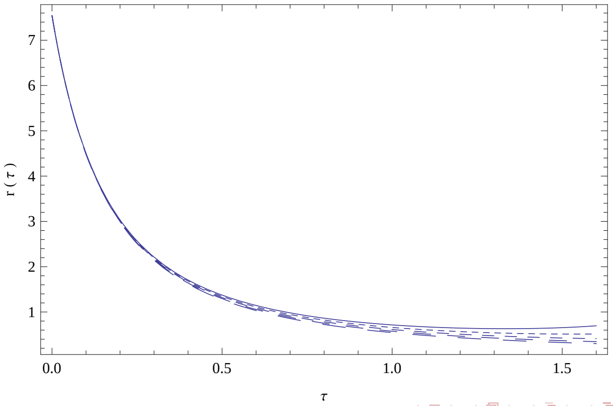
# Dust dominated universe

- The deceleration parameter



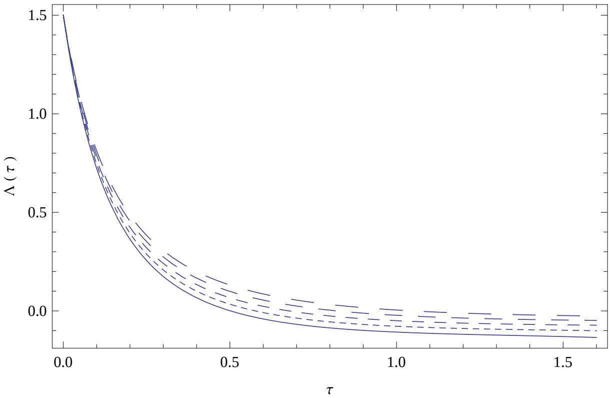
# Dust dominated universe

- The energy density



# Dust dominated universe

- The Lagrange multiplier



# References

Thanks for your attention!

Z. Haghani, T. Harko, H. R. Sepangi, S. Shahidi, arXiv:1404.7689 [gr-qc].

C. Deffayet, G. Esposito-Farese, A. Vikman, Phys. Rev. D 79 (2009) 084003, arXiv:0901.1314 [hep-th].

G. W. Horndeski, J. Math. Phys. 17 (1976) 1980.

A. Chamseddin and V. Mukhanov, JHEP 1311 (2013) 135.

A. Golovnev, arXiv:1310.2790[gr-qc].

Z. Haghani, T. Harko, H. R. Sepangi, S. Shahidi, arXiv:1501.00819 [gr-qc].