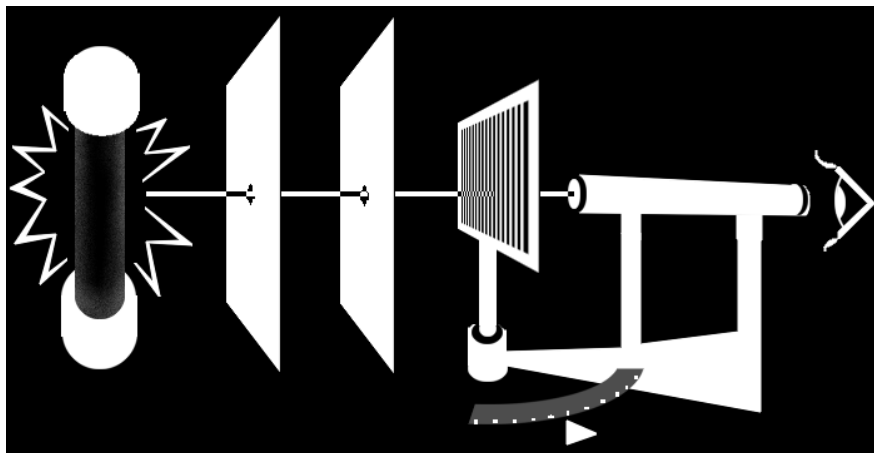


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## Lab 9 - Geometrical Optics



### OBJECTIVES

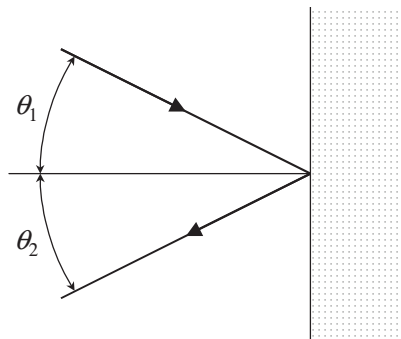
- To examine Snell's Law
- To observe total internal reflection.
- To understand and use the lens equations.
- To find the focal length of a converging lens.
- To discover how lenses form images.
- To observe the relationship between an object and the image formed by a lens.
- To discover how a telescope works

### OVERVIEW

Light is an electromagnetic wave. The theory of the propagation of light and its interactions with matter is by no means trivial; nevertheless, it is possible to understand most of the fundamental features of optical instruments such as eyeglasses, cameras, microscopes, telescopes, etc. through a simple theory based on the idealized concept of a light ray.

A **light ray** is a thin “pencil” of light that travels along a straight line until it encounters matter, at which point it is *reflected*, *refracted*, or *absorbed*. The thin red beam from a laser pointer is a good approximation of such a ray. The study of light rays leads to two important experimental observations:

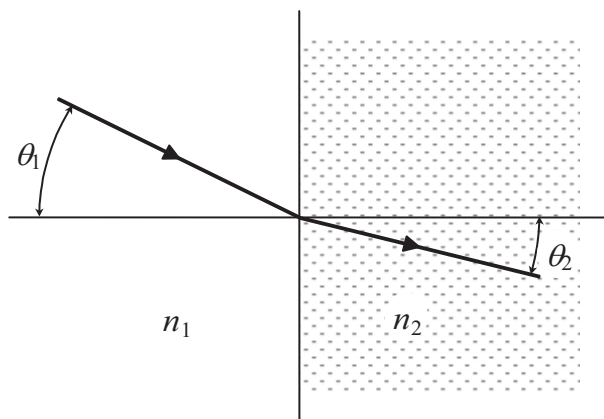
1. When a light ray is reflected by a plane surface, the angle of reflection  $\theta_2$  equals the angle of incidence  $\theta_1$ , as shown in Figure 9.1.

Figure 9.1: Reflection:  $\theta_1 = \theta_2$ 

2. When a light ray travels from one transparent medium into another, as shown in Figure 9.2, the ray is generally “bent” (refracted). The directions of propagation of the incident and refracted rays are related to each other by **Snell’s Law**:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (9.1)$$

where the dimensionless number  $n$  is called the index of refraction and is characteristic of the material.

Figure 9.2: Refraction:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ 

Note that if  $n_1 \sin \theta_1 > n_2$ , no solution is possible for  $\sin \theta_2$ . In this case, none of the light will pass through the interface. *All* of the light will be reflected. This **total internal reflection** is more perfect than reflection by any metallic mirror.

Most transparent materials have indices of refraction between 1.3 and 2.0. The index of refraction of a vacuum is by definition unity. For most purposes, the index of refraction of air ( $n_{\text{air}} = 1.000$ ) can also be taken as unity.

Accurate measurements show the index of refraction to be a function of the wavelength and thus of the color of light. In general, one finds that:

$$n_{\text{blue}} > n_{\text{red}} \quad (9.2)$$

A simplified theoretical explanation of these observations is given by **Huygens' Principle**, which is discussed in elementary physics texts.

**Note:** The room lights will be turned out for these investigations. It will sometimes be difficult to read and write in this manual. Use the desk lamp as needed and good luck. Be patient!

### ***INVESTIGATION 1: Snell's Law***

In this investigation, you will observe and verify Snell's Law by using both a rectangular block and a prism. You will also observe total internal reflection in a prism.

You will need the following materials:

- Rectangular block made of Lucite
- Triangular prism made of Lucite
- Protractor
- Desk lamp
- Light ray box
- Graphing paper from roll (approximately 30 cm for both Activity 1-1 and for Activity 1-2).

#### **Activity 1-1: Verifying Snell's Law**

In this activity, you will verify Snell's Law by using the light ray box with a single ray and the rectangular plastic block.

From Figure 9.3 we can see that Snell's Law and the symmetry of the geometry imply ((assuming  $n_{air} \approx 1$ ):

$$n = \sin \theta_1 / \sin \theta_2 \quad (9.3)$$

$$\theta_1' = \theta_1 \quad (9.4)$$

and

$$s = t \sin(\theta_1 - \theta_2) / \cos \theta_2 \quad (9.5)$$

**WARNING: Please take care not to scratch this block or the other optical elements!**

1. Place the block on a piece of graph paper and align it with the grid. Make sure to place it so that it has at least 10 cm of graph paper on either side. Also **make sure that there is enough room left on the paper for Activity 1-2**. It may help to tape the paper to the table. Trace the outline of the block on the graph paper. Try to use one quadrant of the paper for this activity.

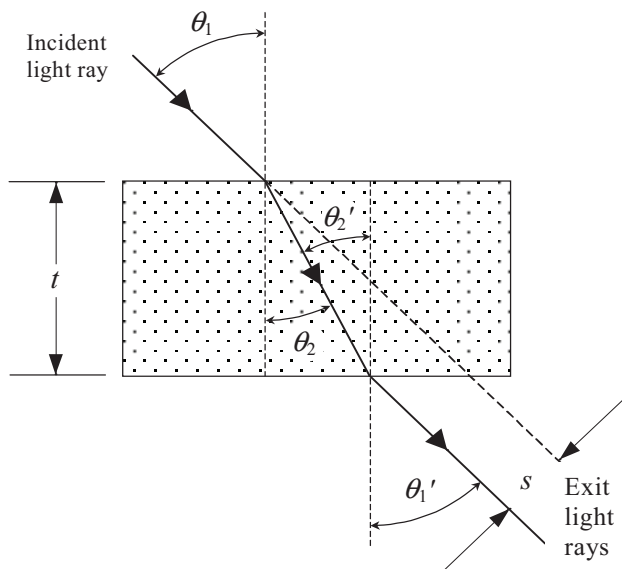


Figure 9.3: Plate with parallel surfaces

**Note:** Only one diagram will be drawn for each group. There are at least three activities that have you draw light rays, so make sure every student does at least one ray tracing diagram.

- Using the single aperture mask, let a single ray from the ray box fall at an oblique angle on the plastic block as in Figure 9.3.

**Hint:** Larger values of  $\theta_1$  produce better results.

- Mark on the graph paper the entry and exit points of the light. Also mark points on the incident and exit rays far from block. This will be necessary to determine the angles.
- After removing the block, trace the light ray paths and use the protractor to measure the angles  $\theta_1, \theta_2, \theta_1'$ , and  $\theta_2'$  (see Figure 9.3). Extrapolate the incident ray so that you can measure  $s$ , the “shift” (or offset) of the output ray relative to the incident ray. Record your results in Table 9.1.

**Note:** We have used the subscript 1 for air and the subscript 2 for plastic, regardless of the direction of the ray. Other conventions are equally valid.

$\theta_1, ^\circ$	$\theta_2, ^\circ$	$\theta_1', ^\circ$	$\theta_2', ^\circ$	$s, \text{ mm}$

Table 9.1:

- Determine the index of refraction  $n$  for the block (see Equation (9.3)). Perhaps have one student begin setting up the next experiment.

$n$ : \_\_\_\_\_

**Question 1-1:** Is Equation (9.4) satisfied? [In other words, are the incident and exit rays parallel?] Discuss.

### Activity 1-2: Light Passing Through a Prism

In this activity, you will study the propagation of light through a prism, as well as observe total internal reflection.

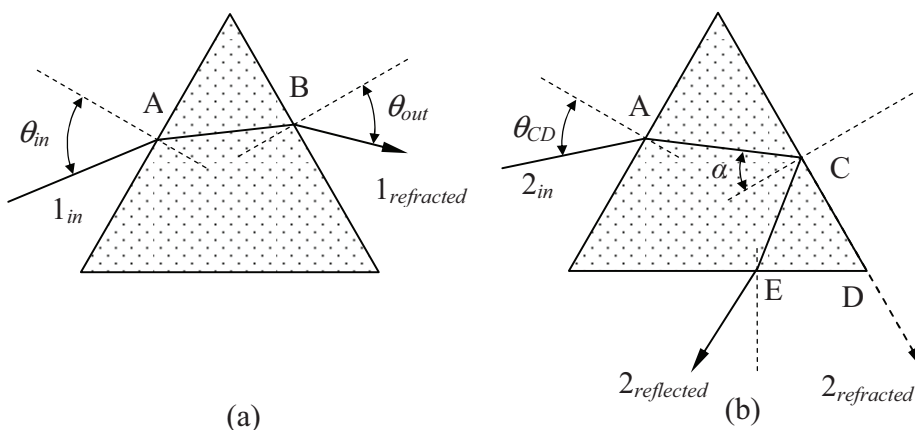


Figure 9.4: Refraction and total reflection in a prism. Follow ray 1 in (a) and ray 2 in (b)

Two examples of light propagation in a prism are shown in Figure 9.4. As you will recall, at each surface some of the light is reflected and some of the light is refracted. Figure 9.4 (a) shows a light ray ( $1_{in}$ ) entering the prism at A at an angle  $\theta_{in}$  (relative to the normal), the refracted ray at the front surface, and the refracted ray ( $1_{refracted}$ ) at the rear surface leaving the prism at B at an angle  $\theta_{out}$ . [Note: At each interface there will also be a reflected ray, but, for clarity, we don't show them here.]

Figure 9.4 (b) shows the case where the refracted output ray ( $2_{refracted}$ , see D) would come out along the edge of the surface ( $\theta_{out} = 90^\circ$ ). Any angle smaller than  $\theta_{CD}$  will produce light that hits the rear surface so that  $n \sin \alpha \geq 1$ , the condition for *total internal reflection*. Such rays are identified by  $2_{reflected}$  and exit at E.

1. To observe this total internal reflection, a triangular prism will be used. Place the prism on a clean area of the graph paper from Activity 1-1 or a new sheet.
2. Set the light ray box so a single light ray falls on one side of the prism.
3. Vary the entrance angle of the ray by slowly rotating the prism. Note that there is a point at which no light is refracted out. Mark the positions of the rays when this *total internal reflection* occurs, as well as trace around the prism. Make sure to mark the incident ray,

the point at which this ray strikes the back of the prism, and the reflected ray once it has exited the prism.

- From these markings and using the protractor, find the internal reflection angle  $\alpha$ . [Hint: Extend lines AC and CD and measure the included angle, which is  $90^\circ - \alpha$ . Alternatively measure the angle ACE with a protractor and divide it by 2 to get the value of  $\alpha$ .]

Angle  $\alpha$ : \_\_\_\_\_

**Question 1-2:** Use Snell's Law to derive an equation for  $n$  in terms of  $\alpha$ . Show your work and calculate  $n$ .

$n$ : \_\_\_\_\_

- Slowly rotate the prism again and note that the exiting ray ( $2_{refracted}$ ) spreads out into various colors just before total internal reflection occurs.

**Question 1-3:** Why does the light spread into different colors prior to total internal reflection?

- Start at the point of total internal reflection and rotate the prism *slightly* to increase the entrance angle. You should see a weak ray just grazing along the outside of the prism base. Note very carefully which color emerges outside the prism first (red or blue).

**Question 1-4:** What color ray emerged first? Discuss what this tells you about the relative magnitudes of  $n_{red}$  and  $n_{blue}$  for Lucite.

## **INVESTIGATION 2: Converging Lenses**

Most optical instruments contain lenses, which are pieces of glass or transparent plastics. To see how optical instruments function, one traces light rays through them. We begin with a simple example by tracing a light ray through a single lens.

We apply Snell's Law to a situation in which a ray of light, coming from a medium with the refractive index  $n_1 = 1$ , e.g. air, falls onto a glass sphere with the index  $n$ , shown in Figure 9.5.

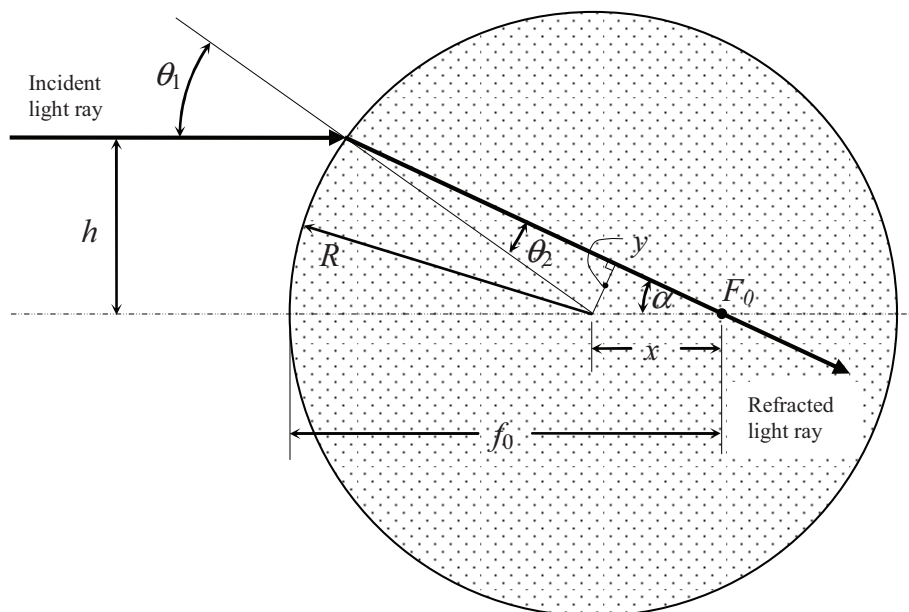


Figure 9.5: Spherical lens

We have in that case

$$\sin \theta_1 = n \sin \theta_2 \quad (9.6)$$

Using the law of sines and Figure 9.5, we obtain

$$\frac{\sin \alpha}{\sin \theta_2} = \frac{y/x}{y/R} = \frac{R}{x} \quad (9.7)$$

or

$$x = R \frac{\sin \theta_2}{\sin \alpha}. \quad (9.8)$$

If we make the simplifying assumption  $h \ll R$ , we can use the approximation

$$\sin \alpha = \frac{h}{\sqrt{(R+x)^2 + h^2}} \approx \frac{h}{R+x}. \quad (9.9)$$

This yields

$$x \approx R \sin \theta_2 \frac{(R+x)}{h} = \frac{R(R+x) \sin \theta_1}{nh} = \frac{R(R+x)h}{nhR}$$

or

$$nx \approx R+x, \quad (9.10)$$

where  $R+x$  is the distance from the front of the sphere to the point  $F$  where the ray crosses the **optical axis**. We call this distance the **focal length**  $f_0$ . Setting  $f_0 = R+x$ , Equation (9.10) now reads:

$$n(f_0 - R) \approx f_0, \quad \text{or} \quad f_0 \approx \frac{nR}{n-1} \quad (9.11)$$

This is a remarkable result because it indicates that, within the limits of our approximation ( $h \ll R$ ), the focal length  $f_0$  is independent of  $h$ . This means that all rays that come in parallel to each other and are close to the axis are collected in one point, the **focal point**,  $F_0$ .

Note that our simple theory of a lens applies only to those cases in which the focal point is inside the sphere. A lens whose focal point is on its inside is not very useful for practical applications; we want it to be on the outside. (Actually, whether inside or out, spheres, for various reasons, do not make very useful lenses.) We will therefore study a more practical lens, the **planoconvex** lens. This lens is bounded on one side by a spherical surface with a radius of curvature  $R$  and on the other by a plane (see Figure 9.6). To keep things simple we make the additional assumption that it is very thin, i.e. that  $d \ll R$ . Now we trace an arbitrary ray that, after having been refracted by the spherical front surface, makes an angle  $\theta_1$  with the optical axis, as shown in Figure 9.6.

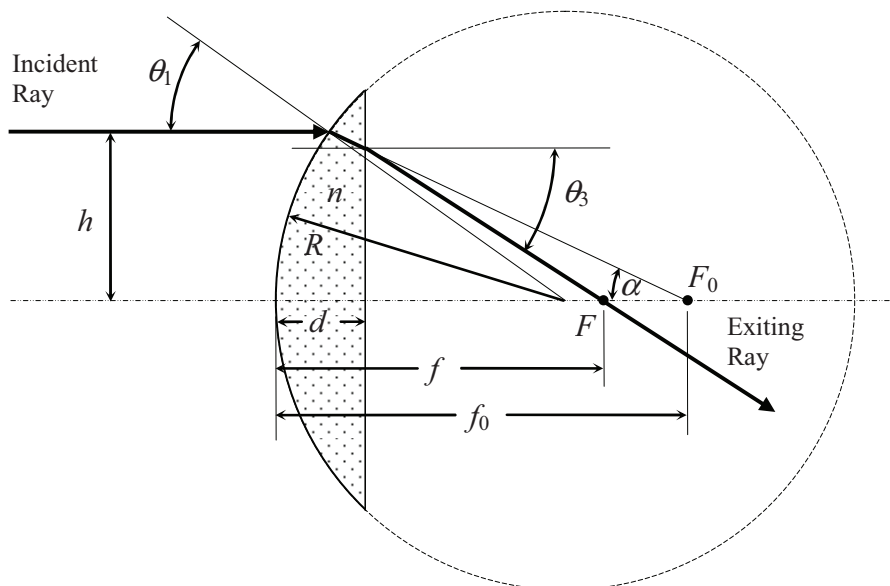


Figure 9.6: Focal point of planoconvex lens

If there were still a full glass sphere, this ray would intersect the optical axis at the point  $F_0$ , a distance  $f_0$  from the front surface. On encountering the planar rear surface of the lens it will instead, according to Snell's Law, be bent to intercept the axis at the point  $F$ , a distance  $f$  from the front. Behind the rear surface is air, so, on the encounter with the second surface Snell's Law becomes:

$$n \sin \alpha = \sin \theta_3 \quad (9.12)$$

But

$$\sin \theta_3 \approx \frac{h}{f} \quad \text{and} \quad \sin \alpha \approx \frac{h}{f_0} \quad (9.13)$$

Hence,



$$f \approx \frac{f_0}{n}, \quad (9.14)$$

i.e. in this case the distance  $f$  is independent of the distance  $h$  (as long as  $h \ll R$  and  $d \ll R$ ). Using Equation (9.11) in Equation (9.14) we find that all incoming rays that are parallel to the optical axis of a *thin* planoconvex lens are collected in a focal point at a distance

$$f = \frac{R}{n-1} \quad (9.15)$$

behind the lens.

What about rays that are not parallel? One can show that all rays issuing from the same object point will be gathered in the same image point (as long as the object is more than one focal length away from the lens).

To find the image point, we only need consider two rays (we'll discuss three that are easy to construct) and find their intersection.

Let us assume that there is a point source of light at the tip of object  $O$  at a distance  $o > f$  in front of the lens. Consider the three rays issuing from this source shown in Figure 9.7.

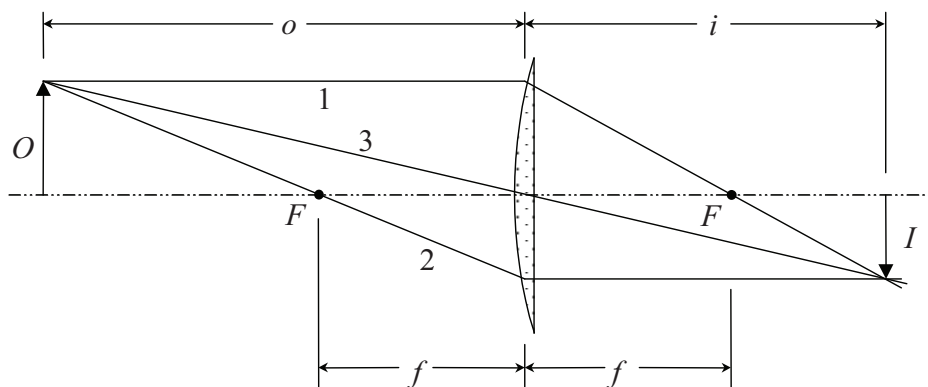


Figure 9.7: Image construction

1. **A ray that is parallel to the axis.** According to what we have just learned, it will go through the focal point  $F$  behind the lens.
2. **A ray that goes through the focal point  $F$  in front of the lens.** With a construction analogous to the one shown in Figure 9.7, one can show that light parallel to the axis coming from behind the lens will go through the focal point in front. Our construction is purely geometrical and cannot depend on the direction of the light beam. We conclude that light that passes through the focal point in front of the lens must leave the lens parallel to the axis. This ray will intersect the first ray at the tip of image  $I$  at a distance  $i$  behind the lens.
3. **A ray that goes through the center of the lens.** At the center, the two glass surfaces are parallel. As we have seen, light passing through such a plate will be shifted by being bent towards the normal at the first interface and then back to the original direction at the second interface. If the plate (in our case the lens) is thin, the shift will be small. We assumed our lens was very thin, so we can neglect any such shift.

From Figure 9.7 it should not be difficult for you to see (from "similar triangles") that:

$$\frac{I+O}{o} = \frac{I}{f} \quad \text{and} \quad \frac{I+O}{i} = \frac{O}{f}$$

Hence we arrive at the following *thin lens formulae*:

$$\text{Thin lens formulae} \quad \frac{1}{f} = \frac{1}{o} + \frac{1}{i} \quad (9.16)$$

$$\frac{i}{I} = \frac{o}{O} \quad (9.17)$$

We define **magnification**  $M$  to be the ratio of the image size  $I$  to the object size  $O$ :

$$M \equiv \frac{I}{O} \quad (9.18)$$

or [by application of Equations (9.16) and (9.17)]:

$$M = \frac{f}{o-f} \quad (9.19)$$

The image in Figure 9.7 is called a **real** image because actual rays converge at the image. The method of image construction used in Figure 9.7, as well as thin lens formulae, can also be formally applied to situations where that is not the case.

What about when the object is *closer* than one focal length? In Figure 9.8, an object  $O$  is placed within the focal distance ( $o < f$ ) of a lens. Following the usual procedure, we draw the ray going through the center of the lens and the one that is parallel to the axis. We add a third ray, originating from  $O$  but going in a direction as if it had come from the first focal point  $F$ . All these are real rays and we draw them as solid lines. We extend the three lines backward as dashed lines and note that all three meet in a single point  $Q$  in front of the lens. To an observer behind the lens, the light coming from  $O$  will *seem* to come from  $Q$  and an upright, magnified image of the object  $O$  will be seen. *This image is a virtual image and not a real image since no light actually issues from  $Q$ .*

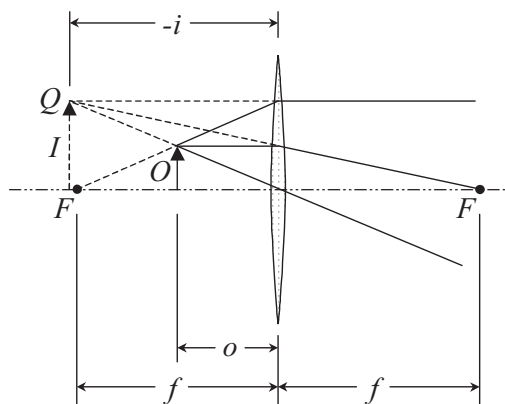


Figure 9.8: Magnifying glass

By an appropriate choice of notation convention, we can apply the thin lens formulae to the magnifying glass. By way of a specific example, setting  $o = f/2$  in these equations, for instance,

yields  $i = -f$ ,  $I = -2O$ , and  $M = -2$ . We interpret the minus sign in the first equation as meaning that  $i$  extends now in *front* of the lens and the minus sign in the second that the image is no longer inverted but upright. We therefore introduce the following convention:  $o$  and  $i$  are taken to be positive if the object is to one side and the image on the other side of the lens.  $O$  and  $I$  are taken to be positive if the object is upright and the image is inverted.

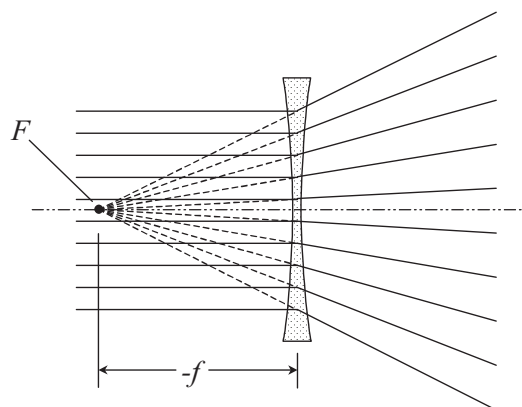


Figure 9.9: Biconcave lens

We can carry this one step further. Concave lenses (lenses that are thinner in the center than on the rim) make parallel incident light diverge. We formally assign to them a negative focal length. Figure 9.9<sup>1</sup> shows that this is justified. To an observer behind such a lens, the incident parallel rays do seem to have come from a virtual focal point in front of the lens.

In this investigation, you will familiarize yourself with a converging lens. You will first find the focal length of the lens and then observe how such a lens creates an image.

For this investigation, you will need the following materials:

- Planoconvex lens made of Lucite
- Light ray box with five ray pattern
- 60 cm of graph paper from roll for focal length activity and the ray tracing

### Activity 2-1: Finding the Focal Length

1. Place the ray box on top of a (new, ~ 60 cm) piece of graph paper. **Make sure that there is enough room left on the graph paper for the next Activity 2-2** Select the five ray pattern by replacing the end piece.
2. In order to do this activity effectively the rays must enter the lens parallel to one another. To adjust the rays, slowly move the top of the box until the rays are parallel with the lines on the graph paper.
3. Place the lens in the center of your graph paper. Let the center ray from the ray box pass through the center of the lens at a  $90^\circ$  angle, as shown in Figure 9.10. Trace the position of the lens on the graph paper.

<sup>1</sup>The lens shown in Figure 9.9 is a biconcave lens; Equations (9.16) and (9.17) apply to it as well, as long as it is thin.

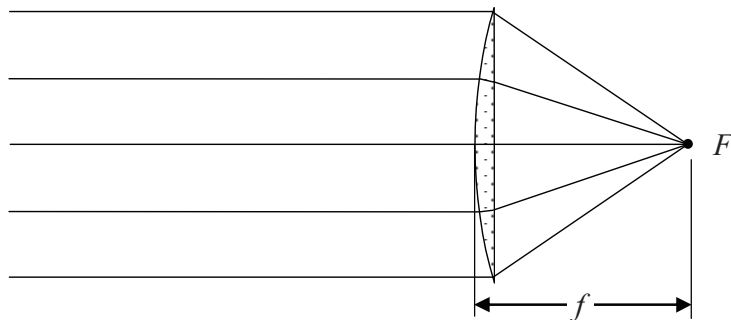


Figure 9.10: Focal point of a planoconvex lens

4. Note that the rays converge at a point on the other side of the lens. This is the focal point  $F$  for the lens. To measure it, make points that will allow you to trace the rays entering and leaving the lens.
5. Remove both the lens and the ray box to measure  $f$ .

$f$ : \_\_\_\_\_ cm

**Prediction 2-1:** What will happen if you place the lens backwards over the position in steps 3 and 4? What will happen to the focal point  $F$  and focal length  $f$ ? *Do not answer before lab.*

6. Turn the lens around and place it at the previous position to determine if the orientation of the lens influences its focal length.

**Question 2-1:** Do the lines converge at the same point as the value that you found in step 4? Should light incident on either side collect at the same point? What does this tell you about the lens?

### Activity 2-2: Ray Tracing

This activity is designed to test the imaging properties of the lens. A ray-tracing diagram like the one shown in Figure 9.7 will be created.

1. Place your 60 cm long piece of graph paper on the table.

2. Align the planoconvex lens somewhere on the graph paper. Allow about 25 cm clear on either side of the lens. Draw the central axis (see Figure 9.7). Draw around the lens to mark its position and mark the two focal points  $F$  on the central axis on either side of the lens. Use the value you found in Activity 2-1.
3. To test how an image is formed, you will draw an object arrow like that shown in Figure 9.7 on your piece of graph paper. Place the tip of the arrow at a distance of  $2f$  from the lens and about 1.0 cm from the central axis. Record your values for the object in Table 9.2.

Object Distance ( $o$ ): \_\_\_\_\_ cm

Object Size ( $O$ ): \_\_\_\_\_ cm

4. Using a single ray from the ray box, mark on your paper the ray paths on both sides of the lens the rays shown in Figure 9.7. Use a different marking scheme ( $\bullet \times o$ ) for points along each of the three rays. Mark two points on either side of the lens to help you draw the rays later after you remove the lens. Your three rays should be as follows:

- Ray 1 should go through  $F$  on its way to the image point
- Ray 2 should enter the lens parallel to the optical axis
- Ray 3 should pass through the lens nearly unbent

Note where the three rays seem to indicate the image should be. You have found the image of only one point – the tip of the object arrow, but that is enough to deduce the entire image. Draw an arrow indicating where the image is. Measure the image distance and image size and fill in the experimental values in Table 9.2. Calculate the magnification (see Equation (9.18)) and enter it into Table 9.2. Include your sheet with your group report.

5. Using the focal length  $f$ , object size  $O$  and object distance  $o$  that you measured above, use the thin lens formulae [Equations (9.16) and (9.17)] to calculate the theoretical values for the image distance and size and the magnification. Insert your calculated values in Table 9.2. Hint: use Equation (9.19) to determine the theoretical magnification.

	Image distance, $i$ (cm)	Image size, $I$ (cm)	Magnification
Experimental			
Theory			

Table 9.2:

**Question 2-2:** Discuss the agreement between your experimental and theoretical values. [Do not be disappointed if things do not work out exactly. Remember that you are not using a truly thin lens.]

### ***INVESTIGATION 3: Image Formation by Converging Lenses***

You will need the following:

- Optical bench
- Lens holder
- 100 mm lens (lens are labeled by focal length)
- 200 mm lens
- Small desk lamp
- Illuminated object
- Viewing screen
- Small see-through ruler
- 3 meter tape

#### **Activity 3-1: Image Formation by a Converging Lens**

In this activity, you will see the relative positions for the object and image distances formed by a converging lens.

**Prediction 3-1:** If the object is always outside of the focal point, do you expect the image distance to increase or decrease if the object distance is increased? **Do this before coming to lab.**

**Prediction 3-2:** What do you expect will happen to the image size if the object distance is increased? **Do before coming to lab.**

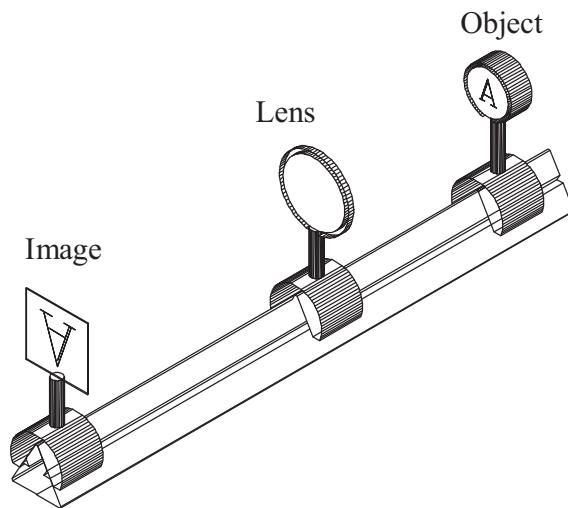


Figure 9.11: Creation of an inverted real image on the optical bench

1. Place an illuminated object together with the mounted 100 mm lens (focal length = 100 mm) and the viewing screen on the optical bench as shown in Figure ??.
2. Measure the size (height) of the object.

Object size: \_\_\_\_\_ cm

3. Set the initial object distance to 15 cm.
4. Find the location of the image. To do this, move the screen until a sharp image is formed. Record the image distance, as well as the image size in the second two columns of Table 9.3.

Object Distance	Image Distance (cm)	Image Size (cm)	Magnification	Upright or Inverted?	Image: Real or Virtual?
15 cm					
20 cm					
30 cm					

Table 9.3: Experimental Data

5. Calculate the magnification of your image and record in Table 9.3.
6. Is the image upright or inverted? Real or virtual? Record your observation in Table 9.3.
7. Try two other object distances, 20 cm and 30 cm. Record the image distance, image size, magnification, orientation and image properties of the image in Table 9.3.

Object distance	Image distance (cm) (pre-lab)	Image size (cm)	Magnification (pre-lab)
15 cm			
20 cm			
30 cm			

Table 9.4: Theoretical Data

8. Use the thin lens formulae to calculate the image distance, image size, and magnification for the three object distances shown in Table 9.4 for the 100 mm lens. Fill columns 2 and 4 of Table 9.4 **before coming to lab**. Enter your calculated values into the table.

**Question 3-1:** How good is the agreement between your experimental data in Table 9.3 and your calculations in Table 9.4? Compare with your Predictions 3-1 and 3-1.

9. Make sure that the object is oriented so it is facing the center of the room and at the end of the optical bench furthest away from the end of the table (nearest the wall). This will make your upcoming observations significantly easier.
10. Place the 10 cm lens so that the object distance is approximately 5-8 cm.
11. Stand at the end of the table so you are looking through the lens at the object (towards the wall). Your distance to the lens should now be approximately 1 m.

**Question 3-2:** Describe your image. Is it upright or inverted? If you were to put a screen where you are looking, would an image form there? What does this tell you about the image? Is it real or virtual?

### Activity 3-2: Simulating a Camera

- Place the object at one end of the optical bench and the viewing screen at the other end.
- Place the 100 mm lens **near** the viewing screen and move the lens until you see a focused image on the screen. (On a real camera, a focus knob will move the lens elements toward or away from the film.) Note the size of the image.
- Repeat with the 200 mm lens (a “telephoto lens” in photography language).



**Question 3-3:** Is the image the same, larger, or smaller?

**Question 3-4:** Based on these results, why are telephoto lenses so long?

### **Activity 3-3: A Telescope**

In this activity, you will see how converging lenses are used in the formation of telescopes.

1. This setup should be somewhere in the lab. You do not need to create it on your optical bench.
2. The 100 mm lens (the eyepiece or ocular) and the 200 mm lens (the objective) should be approximately 30 cm apart on the optical bench.
3. Look through the 100 mm lens (toward the 200 mm lens). You can adjust the distance between the lenses until objects across the room are in sharp focus.

**Question 3-5:** Describe the image you see. Is it upright or inverted? Magnified?

Please clean up your lab area