PHY143 LAB 1: GEOMETRIC OPTICS

Introduction

Optics is the study of the way light interacts with other objects. This behavior can be extremely complicated. However, if the objects in question are much larger than the wavelength of the light being studied, then the light exhibits much simpler behavior. This behavior is described by the laws of geometric optics.

In geometric optics light is assumed to move only in straight lines, except where it meets barriers. At different barriers, light rays are reflected or refracted based on the material properties of the barrier. The angle at which this reflection or refraction occurs is determined by the law of reflection and Snell's law respectively.

In this lab, you will use the technique of ray tracing to verify both the law of reflection and Snell's law. You will then use this same technique to study the properties of lenses (which make use of refraction to focus or defocus light). Finally, you will apply your understanding of lenses to design a small telescope.

THEORY

We will use Fermat's principle of least time to calculate the trajectory of a ray of light for reflection off a flat, smooth surfaces and refraction through a homogeneous medium. First we can place preliminary restrictions on the paths that we consider. For our purposes, we will restrict our attention to two dimensions. Light travels at a constant speed *v* as it travels through a homogeneous medium such as air, glass, or vacuum. We can write *v* in some medium in terms of its proportionality to the speed of light in a vacuum, *c*, by $= \frac{1}{n} c$. We call *n* the *index of refraction* of the medium. Consider a ray of light traveling from point A to point B. Consider

$$v = \frac{s}{t} \Leftrightarrow t = \frac{s}{v} = \frac{n}{c} s$$

where *s* is the total distance traveled and *t* is the total time taken. Since *n* and *c* are constant, we must minimize *s* to minimize *t.* The shortest path between points A and B is a straight line, so this is also the path of least time. Therefore, light travels in a straight line through a continuous medium. This greatly reduces the paths we must consider for more complex situations.

Reflection

Now let's apply this principle for light reflecting off a flat, smooth, surface, e.g. a mirror. Suppose we know light travels from a point A, hits the mirror, and ends up at point C. At what point B does the light hit the mirror? More importantly, since we know light travels in straight lines, how is the angle θ_i of the light ray hitting the surface related to the reflected angle θ_r? We already know the answer, but we will see how this arises from a deeper law.

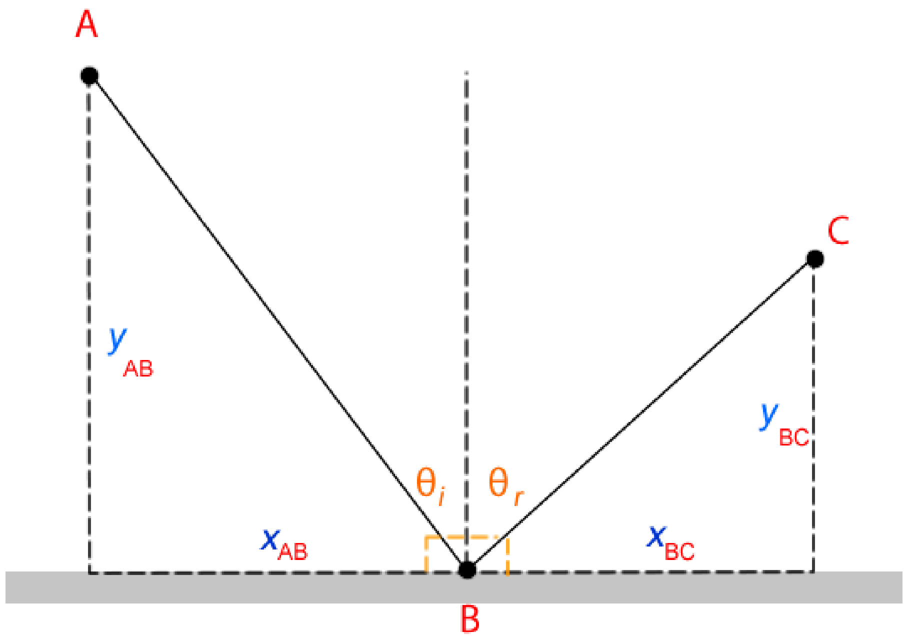


FIGURE 1: REFLECTION

The total time t is given by the sum of times t_{AB} and t_{BC}, the times to travel paths AB and BC respectively. This gives us

$$t = t_{AB} + t_{BC} = \frac{n}{c}(s_{AB} + s_{BC})$$

By the distance formula,

$$s_{AB} = \sqrt{x_{AB}^2 + y_{BA}^2}$$

And

$$s_{BC} = \sqrt{x_{BC}^2 + y_{BC}^2}$$

Then

$$t = \frac{n}{c}\left(\sqrt{x_{AB}^2 + y_{BA}^2} + \sqrt{x_{BC}^2 + y_{BC}^2}\right)$$

Since we only want to vary the position of B along the mirror, we write x_{BC} as $x_{AC} - x_{AB}$:

$$t = \frac{n}{c}\left(\sqrt{x_{AB}^2 + y_{BA}^2} + \sqrt{(x_{AC} - x_{AB})^2 + y_{BC}^2}\right)$$

We then set the derivative of t with respect to x_{AB} to obtain the minimal path:

$$\frac{dt}{dx_{AB}} = \frac{n}{c}\left(\frac{x_{AB}}{\sqrt{x_{AB}^2 + y_{BA}^2}} - \frac{x_{AC} - x_{AB}}{\sqrt{(x_{AC} - x_{AB})^2 + y_{BC}^2}}\right) = 0$$

Substituting in x_{BC} , s_{AB} and s_{BC} we obtain

$$\frac{x_{AB}}{s_{AB}} = \frac{x_{BC}}{s_{BC}}$$

Trigonometry tells us that

$$\frac{x_{AB}}{s_{AB}} = \cos(90° - \theta_i) = \sin\theta_i$$

and

$$\frac{x_{BC}}{s_{BC}} = \cos(90° - \theta_r) = \sin\theta_r$$

so we have

$$\sin\theta_i = \sin\theta_r$$

This implies that, for angles less than 90 degrees as we would assume for a reflection,

$$\theta_i = \theta_r$$

Hence we see that the angle of incidence is equal to the angle of reflection, as expected.

Refraction

Now we consider the path of light as it passes from one medium to another. This means there are two indices of refraction, n_1 and n_2. Suppose light is traveling from a point A in medium 1 to point C in medium 2. At what point B will the light pass through at the barrier between the two mediums and what is the relation between angles θ_1 and θ_2?

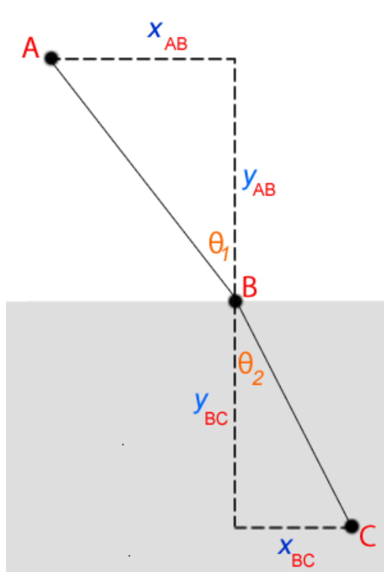


FIGURE 2: REFRACTION

We proceed as we did before:

$$t = t_{AB} + t_{BC} = \frac{1}{c}(n_1 s_{AB} + n_2 s_{BC})$$

Making the same substitution, we obtain

$$t = \frac{1}{c}\left(n_1\sqrt{x_{AB}^2 + y_{BA}^2} + n_2\sqrt{(x_{AC} - x_{AB})^2 + y_{BC}^2}\right)$$

Minimizing yields

$$\frac{dt}{dx_{AB}} = \frac{1}{c}\left(n_1\frac{x_{AB}}{\sqrt{x_{AB}^2 + y_{BA}^2}} - n_2\frac{x_{AC} - x_{AB}}{\sqrt{(x_{AC} - x_{AB})^2 + y_{BC}^2}}\right) = 0$$

Again substituting in x_{BC} , s_{AB} and s_{BC}, this reduces to

$$n_1\frac{x_{AB}}{s_{AB}} = n_2\frac{x_{BC}}{s_{BC}}$$

Trigonometry tells us that $\frac{x_{AB}}{s_{AB}} = \sin\theta_1$ and $\frac{x_{BC}}{s_{BC}} = \sin\theta_2$, so we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

This is Snell's law of refraction.

Many Rays

Now we will look at the properties of many rays passing through optics systems; assemblies of thin concave and convex lenses and mirrors. For simplicity, we will make the *thin lens approximation* and *paraxial approximation*. Before we delve into the meaning of these approximations we will overview the basic concepts of geometric optics.

In the context of geometric optics, an *object(O)* is a source of light that emits the light manipulated in an optical system. For our purposes, objects can be thought of as two dimensional patterns that emit light in all directions from each point.

A *real image (I, Figure 3)* forms in the plane where the light from each point reconvenes. A screen placed in the plane of the real image reveals a crisp projection of the object. Real images are formed by convex lenses and concave mirrors.

A *virtual image (I, Figure 4)* forms in the plane where divergent light appears to converge. Virtual images are formed by concave lenses and convex mirrors, and may also be formed by concave mirrors.

The *focal length* of a lens or mirror is signed distance that characterizes how strongly it focuses light. With the thin lens approximation, the focal point is the distance between the center of the lens and the *focal point (f)*. In the case of a convex lens or concave mirror, the focal point is the point in which collimated (parallel) light is focused or reflected to a single point, and the focal length is positive. In the case of a concave lens or convex mirror, the focal point is the point from which focused or reflected collimated rays appear to diverge, and the focal length is negative.

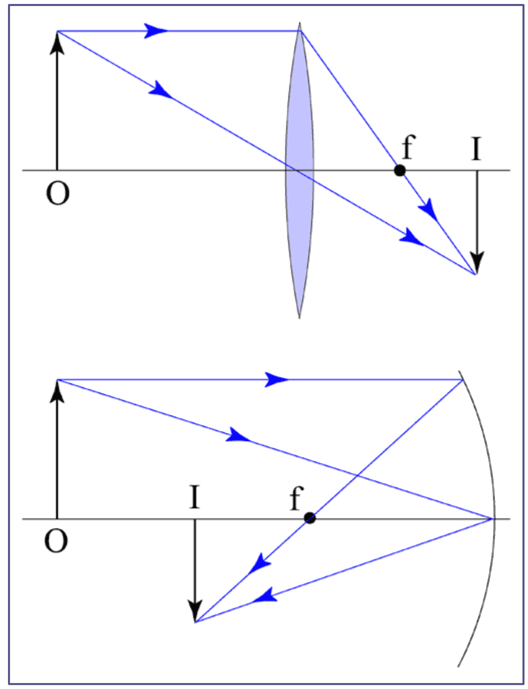


FIGURE 3: REAL IMAGES

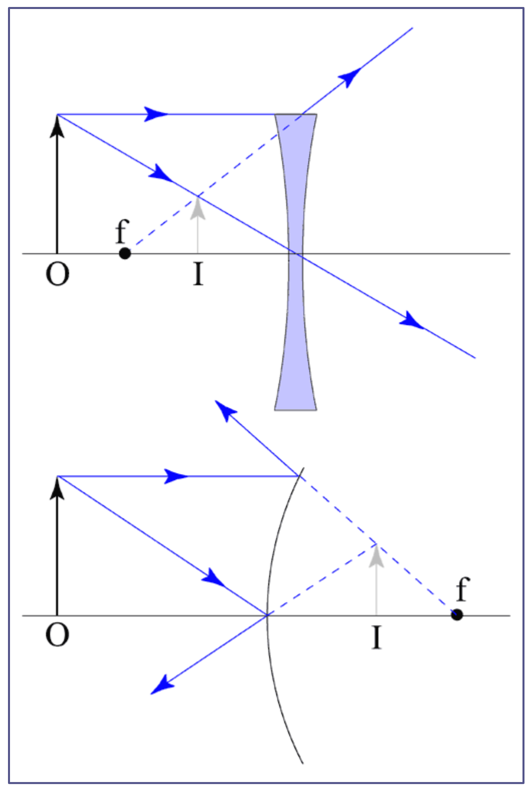


FIGURE 4: VIRTUAL IMAGES

Thin Lens Approximation

The thin rays approximation assumes that the thickness of the lens is negligible compared to the focal length. This allows us to simplify the lensmaker's equation, which relates the refractive index, curvature and thickness of the lens to the focal length. The lensmaker's equation is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{R_1 R_2}\right)$$

Where f is the focal length, n is the refractive index, d is the thickness of the lens, and R_1 and R_2 are the radii of curvature for the front and back of the lens respectively. According to convention, $R_1 < 0 < R_2$ for concave lenses and $R_2 < 0 < R_1$ for convex lenses. If $d \ll f$ then the equation simplifies to

$\frac{1}{f} \cong (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Since the lenses we will be using are symmetrical, we can assume that $R_1 = -R_2$.

Paraxial Approximation

The paraxial approximation assumes that the angles of the rays are small with respect to the *optical axis* (the central path along which light travels). This allows us to make a simple relation between the radius of curvature R of the concave and convex mirrors to the focal length f, given by

$$R = -2f$$

Here we assume that the radius of curvature is positive for convex mirrors and negative for concave mirrors.

Combined with the thin lens approximation, the paraxial approximation yields a simple relation between the focal length of a lens and the positions of the object and image, given by

$$\frac{1}{f} = \frac{1}{d_O} + \frac{1}{d_i}$$

Where f is the focal length, d_o is the displacement between the object and the lens, and d_i is the displacement between the image and the lens. According to convention, the focal length is positive for convex lenses and concave mirrors, while it is negative for concave lenses and convex mirrors.

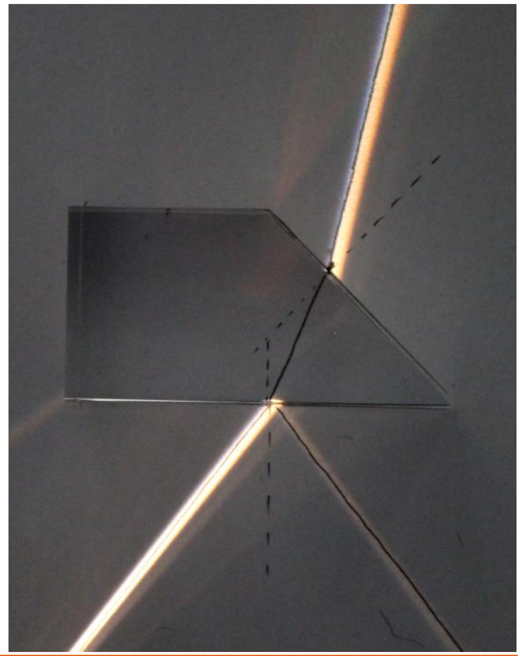
The magnification of the image is given by

$$M = \frac{d_i}{d_o}$$

Where positive M denotes an erect (upright) image and negative M denotes an inverted (upside-down) image.

EXPERIMENT

Ray Tracing



To trace a ray:

1) Turn on the Pasco light source and select either one slit or multiple slits, depending on the measurement you are making.

2) Place the object you are measuring on a piece of paper in the path of the light beam. Trace both the edges of the object and the front of the light box so that you can reset them if you accidently move them.

3) Trace the beam on the paper using a pencil. This may be easiest to do by making dots in the center of the beam where it hits the prism and far away, and then later connecting these dots with a ruler.

Unlike a mathematical ray, the beam from the light box is not infinitely thin. What errors will this introduce, and how can you minimize them?

Experiments

1) Verify the law of reflection for a plane mirror.
2) Verify Snell's law for a prism.
3) Determine the focal length of one of the concave lenses and one of the convex lenses by setting the light source to produce multiple rays of light at once.
4) Design a telescope on the optics bench with a magnification of 2x.

- How accurately can you trace the rays? How accurately can you measure the angles between them?
- How accurately can you determine where two rays intersect?