The following example contains the full description of the solution of one of the examples from the paper entitled "Applying Gröbner Basis Method to Multiparametric Polynomial Nonlinear Programming" authored by B. M.-Alizadeh, A. Basiri and S. Rahmany.

Example 4.8. Consider the following Mp-NLP problem with 3 parameters, 2 variables and 10 constraints.

$$\begin{array}{rl} Minimum & 2x_2^2 + \frac{1}{2}p_1x_2 + x_1x_2 + \frac{21}{16}p_1^2 + \frac{5}{4}x_1p_1 + \frac{9}{4}x_1^2 \\ subject\ to & p_2 - 5 = \frac{1}{2}p_1 + x_1 - 5 \leq 0 \\ & -p_2 - 5 = -\frac{1}{2}p_2 + x_1 - 5 \leq 0 \\ & p_3 - 5 = \frac{1}{2}p_2 + x_2 - 5 = \frac{1}{2}(\frac{1}{2}p_1 + x_1) + x_2 - 5 \leq 0 \\ & -p_3 - 5 = -\frac{1}{2}p_2 + x_2 - 5 = -\frac{1}{2}(\frac{1}{2}p_1 + x_1) + x_2 - 5 \leq 0. \end{array}$$

An eigenvalue system for this problem is computed which contains 6 elements as follows.

$$E_1 = \{-\tfrac{3}{2}p_3p_1 + \tfrac{25}{16}p_1^2 - \tfrac{9}{8}p_2p_1 + \tfrac{13}{16}p_2^2 + \tfrac{3}{2}p_3p_2 + 2p_3^2, -\tfrac{2}{9}p_3p_1 + \tfrac{41}{36}p_1^2 + \tfrac{1}{9}p_2p_1 + \tfrac{17}{36}p_2^2 - \tfrac{17}{9}p_3p_2 + \tfrac{17}{9}p_3^2,$$
$$\tfrac{61}{52}p_1^2 - \tfrac{6}{13}p_3p_1 + \tfrac{17}{13}p_3^2, -3p_3p_2 + \tfrac{5}{4}p_1^2 - p_2p_1 + \tfrac{13}{4}p_2^2 + 2p_3^2, -p_3p_2 + \tfrac{5}{4}p_1^2 - p_2p_1 + \tfrac{9}{4}p_2^2 + 2p_3^2,$$
$$-\tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{5}{8}p_2p_1 + \tfrac{9}{16}p_2^2 + \tfrac{1}{2}p_3p_2 + 2p_3^2, -\tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{3}{8}p_2p_1 + \tfrac{13}{16}p_2^2 - \tfrac{3}{2}p_3p_2 + 2p_3^2,$$
$$\tfrac{77}{68}p_1^2, \tfrac{5}{4}p_1^2 - p_2p_1 + \tfrac{17}{8}p_2^2, \tfrac{5}{4}p_1^2 - 2p_3p_1 + 9p_3^2, \tfrac{41}{32}p_1^2 - \tfrac{9}{16}p_2p_1 + \tfrac{17}{32}p_2^2, \tfrac{41}{36}p_1^2 - \tfrac{2}{9}p_3p_1 + \tfrac{17}{9}p_3^2,$$
$$-4p_3p_1 + \tfrac{5}{4}p_1^2 + p_2p_1 + \tfrac{9}{4}p_2^2 - 17p_3p_2 + 34p_3^2, -4p_3p_1 + \tfrac{5}{4}p_1^2 + p_2p_1 + \tfrac{13}{4}p_2^2 - 23p_3p_2 + 42p_3^2,$$
$$\tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{7}{8}p_2p_1 + \tfrac{21}{16}p_2^2 - \tfrac{5}{2}p_3p_2 + 2p_3^2, \tfrac{2}{9}p_3p_1 + \tfrac{41}{36}p_1^2 - \tfrac{1}{9}p_2p_1 + \tfrac{17}{36}p_2^2 - \tfrac{17}{9}p_3p_2 + \tfrac{17}{9}p_3^2\},$$

where

$$N_1 = \{\ \},\ W_1 = \{p_2 - 2p_3, p_1 - 3p_2, p_1 - p_2 - 4p_3, p_1 + p_2 - 8p_3\}.$$

$$E_2 = \{\tfrac{21}{16}p_1^2 - \tfrac{147}{136}p_2p_1 + \tfrac{441}{272}p_2^2, \tfrac{21}{16}p_1^2 - \tfrac{3}{4}p_2p_1 + \tfrac{13}{16}p_2^2, -\tfrac{2}{9}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{67}{72}p_2p_1 + \tfrac{293}{144}p_2^2 - \tfrac{17}{9}p_3p_2 + \tfrac{17}{9}p_3^2,$$
$$\tfrac{5}{26}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{123}{104}p_2p_1 + \tfrac{477}{208}p_2^2 - \tfrac{51}{26}p_3p_2 + \tfrac{17}{13}p_3^2, \tfrac{9}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{23}{8}p_2p_1 + \tfrac{213}{16}p_2^2 - \tfrac{85}{2}p_3p_2 + 34p_3^2,$$
$$\tfrac{11}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{27}{8}p_2p_1 + \tfrac{253}{16}p_2^2 - \tfrac{103}{2}p_3p_2 + 42p_3^2, \tfrac{13}{18}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{29}{24}p_2p_1 + \tfrac{33}{16}p_2^2 - \tfrac{17}{6}p_3p_2 + \tfrac{17}{9}p_3^2,$$
$$\tfrac{2}{9}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{83}{72}p_2p_1 + \tfrac{293}{144}p_2^2 - \tfrac{17}{9}p_3p_2 + \tfrac{17}{9}p_3^2, \tfrac{5}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{15}{8}p_2p_1 + \tfrac{81}{16}p_2^2 - \tfrac{27}{2}p_3p_2 + 9p_3^2,$$
$$-\tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{3}{8}p_2p_1 + \tfrac{13}{16}p_2^2 - \tfrac{3}{2}p_3p_2 + 2p_3^2, \tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{7}{8}p_2p_1 + \tfrac{21}{16}p_2^2 - \tfrac{5}{2}p_3p_2 + 2p_3^2\},$$

where

$$N_2 = \{p_1 - 3p_2\},\ W_2 = \{p_2 - 2p_3\}.$$

$$E_3 = \{-5p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{7}{4}p_2p_1 + \tfrac{77}{16}p_2^2 - 26p_3p_2 + 36p_3^2, -3p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{11}{24}p_2p_1 + \tfrac{31}{48}p_2^2 - \tfrac{14}{3}p_3p_2 + 13p_3^2,$$
$$-\tfrac{77}{26}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{41}{104}p_2p_1 + \tfrac{53}{208}p_2^2 - \tfrac{89}{26}p_3p_2 + \tfrac{161}{13}p_3^2, -\tfrac{49}{17}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{49}{136}p_2p_1 + \tfrac{49}{272}p_2^2 - \tfrac{49}{17}p_3p_2 + \tfrac{196}{17}p_3^2,$$
$$-\tfrac{23}{9}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{17}{72}p_2p_1 + \tfrac{31}{48}p_2^2 - \tfrac{14}{3}p_3p_2 + 13p_3^2, -\tfrac{11}{2}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{17}{8}p_2p_1 + \tfrac{101}{16}p_2^2 - \tfrac{65}{2}p_3p_2 + 42p_3^2,$$
$$-\tfrac{9}{2}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{13}{8}p_2p_1 + \tfrac{77}{16}p_2^2 - \tfrac{51}{2}p_3p_2 + 34p_3^2, -\tfrac{5}{2}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{5}{8}p_2p_1 + \tfrac{9}{16}p_2^2 - \tfrac{9}{2}p_3p_2 + 9p_3^2,$$
$$-\tfrac{5}{2}p_3p_1 + \tfrac{21}{16}p_1^2 + \tfrac{29}{72}p_2p_1 + \tfrac{11}{48}p_2^2 - \tfrac{49}{18}p_3p_2 + 9p_3^2, -\tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{3}{8}p_2p_1 + \tfrac{13}{16}p_2^2 - \tfrac{3}{2}p_3p_2 + 2p_3^2,$$
$$-\tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{1}{2}p_2p_1 + \tfrac{9}{16}p_2^2 - \tfrac{1}{2}p_3p_2 + 2p_3^2, \tfrac{1}{2}p_3p_1 + \tfrac{21}{16}p_1^2 - \tfrac{7}{8}p_2p_1 + \tfrac{21}{16}p_2^2 - \tfrac{5}{2}p_3p_2 + 2p_3^2\},$$

where

$$N_3 = \{p_1 + p_2 - 8p_3\},\ W_3 = \{p_2 - 2p_3, p_1 - 3p_2, p_1 - p_2 - 4p_3\}.$$

$$E_4 = \{\tfrac{21}{16}p_1^2 - \tfrac{147}{68}p_3p_1 + \tfrac{441}{68}p_3^2, \tfrac{21}{16}p_1^2 - \tfrac{113}{52}p_3p_1 + \tfrac{341}{52}p_3^2, \tfrac{21}{16}p_1^2 - \tfrac{61}{36}p_3p_1 + \tfrac{161}{36}p_3^2, \tfrac{21}{16}p_1^2 - \tfrac{25}{12}p_3p_1 + \tfrac{25}{4}p_3^2,$$
$$\tfrac{21}{16}p_1^2 - \tfrac{5}{4}p_3p_1 + \tfrac{9}{4}p_3^2, \tfrac{21}{16}p_1^2 - \tfrac{3}{2}p_3p_1 + \tfrac{13}{4}p_3^2\},$$

where

$$N_4 = \{p_1 - 6p_3, p_2 - 2p_3\},\ W_4 = \{\}.$$

$E_5 = \{\frac{9}{16}p_2^2 - \frac{5}{8}p_2p_1 + \frac{21}{16}p_1^2, -\frac{7}{6}p_3p_1 + \frac{21}{16}p_1^2 - \frac{11}{24}p_2p_1 + \frac{31}{48}p_2^2 - \frac{1}{2}p_3p_2 + \frac{14}{3}p_3^2,$
$-\frac{18}{13}p_3p_1 + \frac{21}{16}p_1^2 - \frac{41}{104}p_2p_1 + \frac{53}{208}p_2^2 + \frac{18}{13}p_3p_2 + \frac{36}{13}p_3^2, -\frac{8}{9}p_3p_1 + \frac{21}{16}p_1^2 - \frac{29}{72}p_2p_1 + \frac{11}{48}p_2^2 + \frac{8}{9}p_3p_2 + \frac{16}{9}p_3^2,$
$-\frac{49}{34}p_3p_1 + \frac{21}{16}p_1^2 - \frac{49}{136}p_2p_1 + \frac{49}{272}p_2^2 + \frac{49}{34}p_3p_2 + \frac{49}{17}p_3^2, -\frac{29}{18}p_3p_1 + \frac{21}{16}p_1^2 - \frac{17}{72}p_2p_1 + \frac{31}{48}p_2^2 - \frac{1}{2}p_3p_2 + \frac{14}{3}p_3^2,$
$2p_3p_1 + \frac{21}{16}p_1^2 - \frac{13}{8}p_2p_1 + \frac{77}{16}p_2^2 - 13p_3p_2 + 9p_3^2, 3p_3p_1 + \frac{21}{16}p_1^2 - \frac{17}{8}p_2p_1 + \frac{101}{16}p_2^2 - 18p_3p_2 + 13p_3^2,$
$-3p_3p_1 + \frac{21}{16}p_1^2 + \frac{7}{8}p_2p_1 + \frac{21}{16}p_2^2 - 8p_3p_2 + 13p_3^2, -2p_3p_1 + \frac{21}{16}p_1^2 + \frac{3}{8}p_2p_1 + \frac{13}{16}p_2^2 - 5p_3p_2 + 9p_3^2,$
$-\frac{5}{2}p_3p_1 + \frac{21}{16}p_1^2 + \frac{1}{2}p_2p_1 + \frac{9}{16}p_2^2 - 4p_3p_2 + 9p_3^2, -\frac{1}{2}p_3p_1 + \frac{21}{16}p_1^2 - \frac{3}{8}p_2p_1 + \frac{13}{16}p_2^2 - \frac{3}{2}p_3p_2 + 2p_3^2,$
$-\frac{1}{4}p_3p_1 + \frac{21}{16}p_1^2 - \frac{5}{8}p_2p_1 + \frac{9}{16}p_2^2 + \frac{1}{4}p_3p_2 + \frac{1}{2}p_3^2, \frac{1}{2}p_3p_1 + \frac{21}{16}p_1^2 - \frac{7}{8}p_2p_1 + \frac{21}{16}p_2^2 - \frac{5}{2}p_3p_2 + 2p_3^2\},$

where

$$N_5 = \{p_1 - p_2 - 4p_3\},\ W_5 = \{p_2 - 2p_3, p_1 - 3p_2\}.$$

$E_6 = \{\frac{41}{36}p_1^2, \frac{77}{68}p_1^2, \frac{5}{4}p_1^2 - 2p_3p_1 + 9p_3^2, \frac{5}{4}p_1^2 - 2p_3p_1 + \frac{17}{2}p_3^2, \frac{21}{16}p_1^2 - \frac{7}{4}p_3p_1 + \frac{21}{4}p_3^2, \frac{21}{16}p_1^2 - \frac{5}{4}p_3p_1 + \frac{9}{4}p_3^2,$
$\frac{25}{16}p_1^2 - \frac{15}{4}p_3p_1 + \frac{33}{4}p_3^2, \frac{41}{32}p_1^2 - \frac{9}{8}p_3p_1 + \frac{17}{8}p_3^2, \frac{41}{36}p_1^2 - \frac{2}{9}p_3p_1 + \frac{17}{9}p_3^2, \frac{61}{52}p_1^2 - \frac{6}{13}p_3p_1 + \frac{17}{13}p_3^2\},$

where

$$N_6 = \{p_2 - 2p_3\},\ W_6 = \{p_1 - 6p_3\}.$$