فصل پنج:

طراحی و تحلیل تیرها برای خمش

Chapter 5:

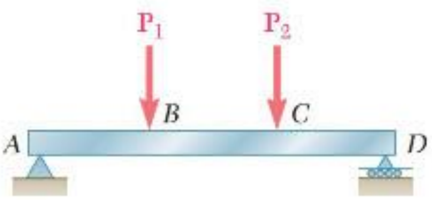
Analysis and design of beams for bending

Introduction + مقدمه

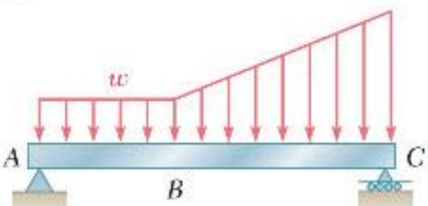
+ جنس تیرهای رایج در مهندسی سازه و مهندسی مکانیک {
فولاد steel
آلومینیوم Aluminum

+ بارهای اعمالی به تیرها موارد زیر را ایجاد {
برش shear
خمش bending
نیروی محوری Axial force

+ انواع بارهای اعمالی به تیرها
→ بار متمرکز Concentrated force — Newtons, pounds, kilonewtons, kips
→ بار گسترده distributed force — N/m, KN/m, lb/ft, kips/ft
→ ترکیب بار متمرکز و بار گسترده

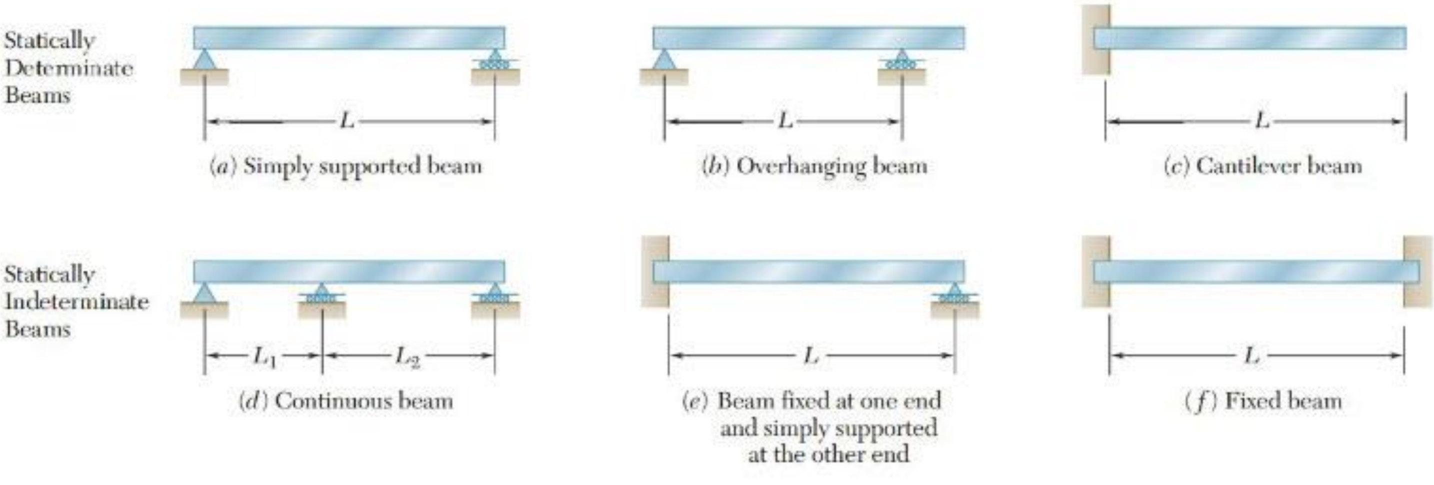


(a) Concentrated loads



(b) Distributed loads

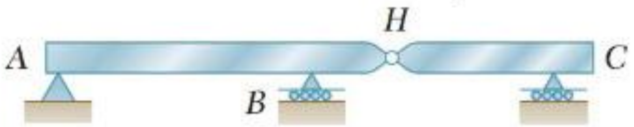
+ دسته‌بندی تیرها براساس نوع تکیه‌گاه‌شان
→ از نظر استاتیکی معین statically determinate
→ از نظر استاتیکی نامعین statically indeterminate
← در فصل ۹ بررسی خواهد شد



(a) Simply supported beam
(b) Overhanging beam
(c) Cantilever beam
(d) Continuous beam
(e) Beam fixed at one end and simply supported at the other end
(f) Fixed beam

• بعضی وقت‌ها، دو یا چند تیر توسط پین به یکدیگر متصل می‌شوند. برای تحلیل این نوع تیرها، از این نکته استفاده می‌شود که گشتاور خمشی در محل پین صفر است. بنابراین با بررسی جداگانه هر تیر، مجهولات قابل تعیین می‌باشند.

(a)

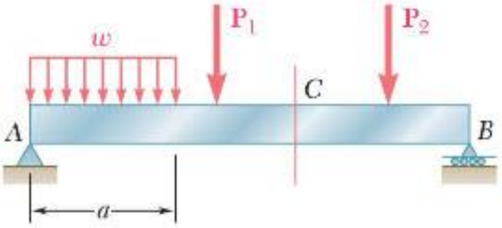


(b)

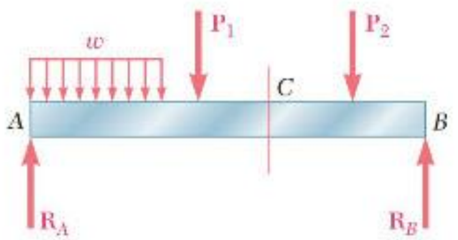
• تاثیر بارگذاری‌های متفاوت در نیروهای درونی تیرها ← بارهای عرضی متمرکز و گسترده / گشتاور خمشی

الف) بار عرضی متمرکز و گسترده سبب ایجاد نیروهای درونی در هر مقطع تیر می‌شوند که عبارتند از نیروی برشی V و گشتاور خمشی M.

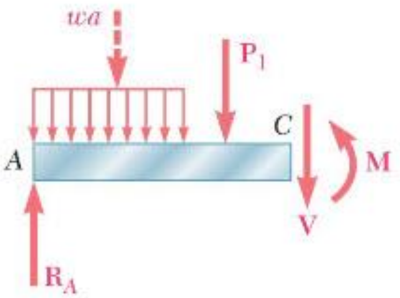
• نحوه تحلیل
- رسم دیاگرام آزاد کل تیر برای بدست آوردن عکس‌العمل تکیه‌گاه‌ها
- زدن مقطع در محل مورد نظر و رسم مجدد دیاگرام آزاد برای یافتن نیروهای درونی



(*a*) Transversely-loaded beam



(*b*) Free-body diagram to find support reactions



(*c*) Free-body diagram to find internal forces at *C*

نیروی برشی V سبب ایجاد تنش برشی می‌شود
گشتاور خمشی M سبب ایجاد تنش محوری می‌شود.

نکته: معمولاً تنش محوری ناشی از گشتاور خمشی تاثیر غالب دارد نسبت به تنش برشی ناشی از نیروی برشی. به همین دلیل، در طراحی‌ها معمولاً از اثر تنش برشی صرف‌نظر می‌شود.

ناشی از نیروی برشی نه ناشی از گشتاور خمشی

• در طراحی تیرها در نظر گرفتن دو نکته زیر مفید می‌باشد

۱) اگر سطح مقطع تیر، متغیر باشد آن‌گاه طبق رابطه $\sigma_m = \frac{|M|}{S}$ ، مقطعی برای تیر انتخاب می‌شود که دارای S بزرگی باشد. (مقدار S برخی تیرها در پیوست C آورده شده است.)
برای تیرهای با مقطع مستطیلی، $S = \frac{1}{6}bh^2$ است.

۲) از آنجا که معمولا سطح مقطع تیرها یکنواخت است نکته‌ای که در طراحی مدنظر قرار می‌گیرد این است که بیشترین مقدار تنش خمشی به ازای بیشترین مقدار گشتاور خمشی اتفاق می‌افتد.
بنابراین وظیفه اساسی در طراحی تیرها، پیدا کردن محل و مقدار بیشترین گشتاور خمشی در تیر است.
این امر با رسم نمودار گشتاور خمشی Bending-moment diagram امکان‌پذیر می‌باشد.

Shear and bending-moment diagrams

دیاگرام‌های گشتاور خمشی و نیروی برشی

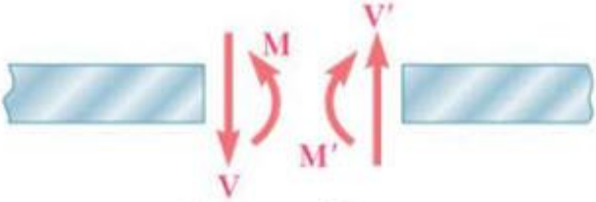
قبل از هر چیزی، قرارداد علامت نیروی برشی و گشتاور خمشی بیان می‌شود:

1) اگر نیروی برشی و گشتاور خمشی به صورت نیروهای درونی در نظر باشند آن‌گاه آنها را مثبت در نظر می‌گیریم به شرطی که مانند شکل (a) باشند.

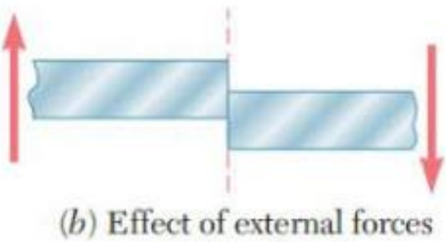
2) اگر نیروی برشی و گشتاور خمشی به صورت نیروهای خارجی در نظر باشند آن‌گاه

الف) نیروی برشی را زمانی مثبت در نظر می‌گیریم که نیروهای خارجی (شامل نیروها و عکس‌العمل‌ها) وارده بر تیر، تمایل به برش تیر مشابه شکل (b) داشته باشند.

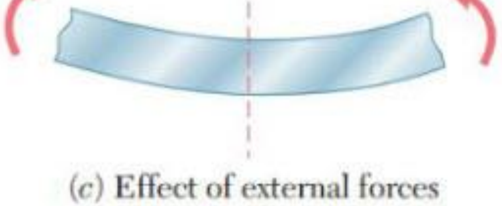
ب) گشتاور خمشی را زمانی مثبت در نظر می‌گیریم که نیروهای خارجی وارده بر تیر، تمایل به خم کردن تیر مشابه شکل (c) داشته باشند.



(*a*) Internal forces
(positive shear and positive bending moment)



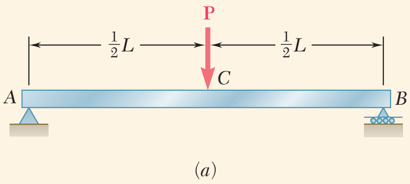
(*b*) Effect of external forces
(positive shear)



(*c*) Effect of external forces
(positive bending moment)

Concept Application 5.1

Draw the shear and bending-moment diagrams for a simply supported beam AB of span L subjected to a single concentrated load **P** at its midpoint C (Fig. 5.7a).



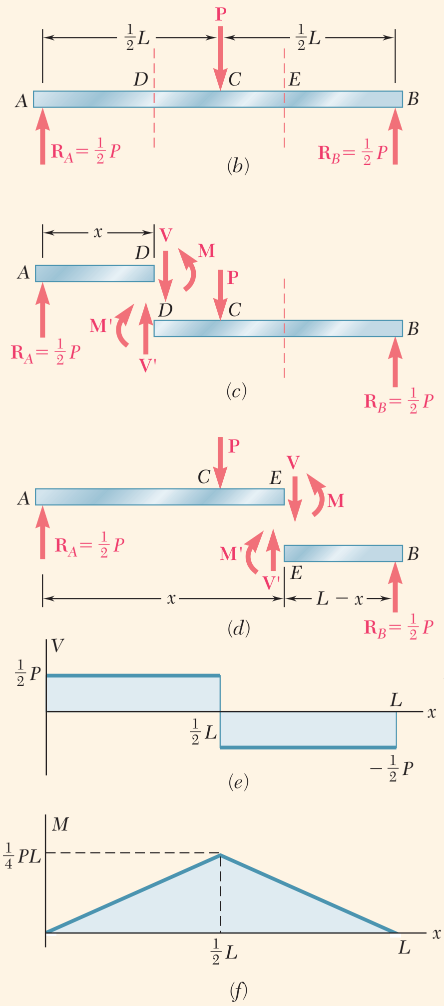


Fig. 5.7 (*a*) Simply supported beam with midpoint load, **P.** (*b*) Free-body diagram of entire beam. (*c*) Free-body diagrams with section taken to left of load **P.** (*d*) Free-body diagrams with section taken to right of load **P.** (*e*) Shear diagram. (*f*) Bending-moment diagram.

Determine the reactions at the supports from the free-body diagram of the entire beam (Fig. 5.7*b*). The magnitude of each reaction is equal to $P/2$.

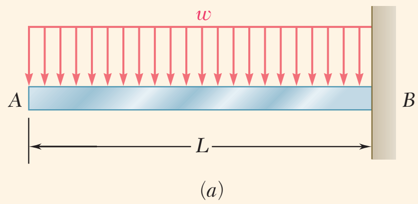
Next cut the beam at a point D between A and C and draw the free-body diagrams of AD and DB (Fig. 5.7*c*). *Assuming that the shear and bending moment are positive,* we direct the internal forces **V** and **V**′ and the internal couples **M** and **M**′ as in Fig. 5.6*a*. Consider the free body AD. The sum of the vertical components and the sum of the moments about D of the forces acting on the free body are zero, so $V = +P/2$ and $M = +Px/2$. Both the shear and the bending moment are positive. This is checked by observing that the reaction at A tends to shear off and bend the beam at D as indicated in Figs. 5.6*b* and *c*. We plot V and M between A and C (Figs. 5.8*d* and *e*). The shear has a constant value $V = P/2$, while the bending moment increases linearly from $M = 0$ at $x = 0$ to $M = PL/4$ at $x = L/2$.

Cutting the beam at a point E between C and B and considering the free body EB (Fig. 5.7*d*), the sum of the vertical components and the sum of the moments about E of the forces acting on the free body are zero. Obtain $V = -P/2$ and $M = P(L - x)/2$. Therefore, the shear is negative, and the bending moment positive. This is checked by observing that the reaction at B bends the beam at E as in Fig. 5.6*c* but tends to shear it off in a manner opposite to that shown in Fig. 5.6*b*. The shear and bending-moment diagrams of Figs. 5.7*e* and *f* are completed by showing the shear with a constant value $V = -P/2$ between C and B, while the bending moment decreases linearly from $M = PL/4$ at $x = L/2$ to $M = 0$ at $x = L$.

Note from the previous Concept Application that when a beam is subjected only to concentrated loads, the shear is constant between loads and the bending moment varies linearly between loads. In such situations, the shear and bending-moment diagrams can be drawn easily once the values of V and M have been obtained at sections selected just to the left and just to the right of the points where the loads and reactions are applied (see Sample Prob. 5.1).

Concept Application 5.2

Draw the shear and bending-moment diagrams for a cantilever beam AB of span L supporting a uniformly distributed load w (Fig. 5.8*a*).



Cut the beam at a point C, located between A and B, and draw the free-body diagram of AC (Fig. 5.8*b*), directing **V** and **M** as in Fig. 5.6*a*. Using the distance x from A to C and replacing the distributed load over AC by its resultant wx applied at the midpoint of AC, write

$$+\uparrow \Sigma F_y = 0: \qquad -wx - V = 0 \qquad V = -wx$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad wx\left(\frac{x}{2}\right) + M = 0 \qquad M = -\frac{1}{2}wx^2$$

Note that the shear diagram is represented by an oblique straight line (Fig. 5.8*c*) and the bending-moment diagram by a parabola (Fig. 5.8*d*). The maximum values of V and M both occur at B, where

$$V_B = -wL \qquad M_B = -\tfrac{1}{2}wL^2$$

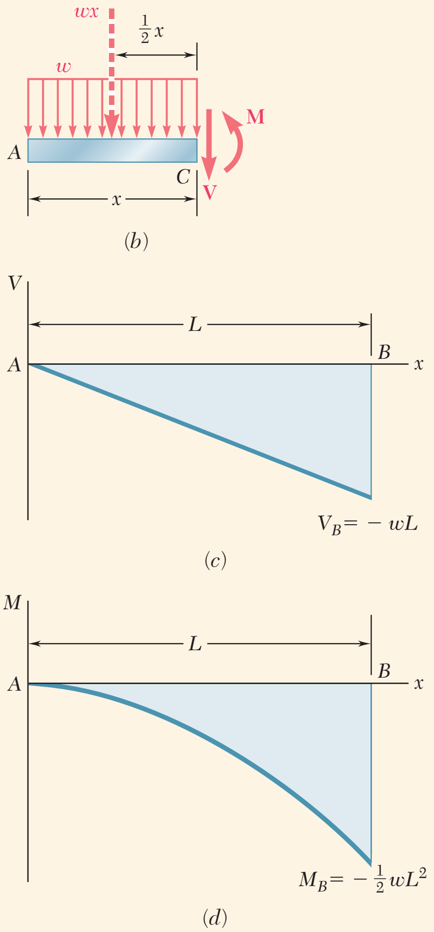
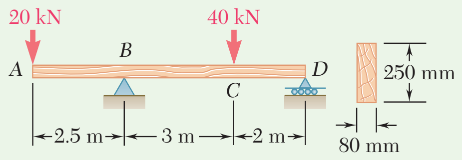


Fig. 5.8 (*a*) Cantilevered beam supporting a uniformly distributed load. (*b*) Free-body diagram of section *AC*. (*c*) Shear diagram. (*d*) Bending-moment diagram.

Sample Problem 5.1

For the timber beam and loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



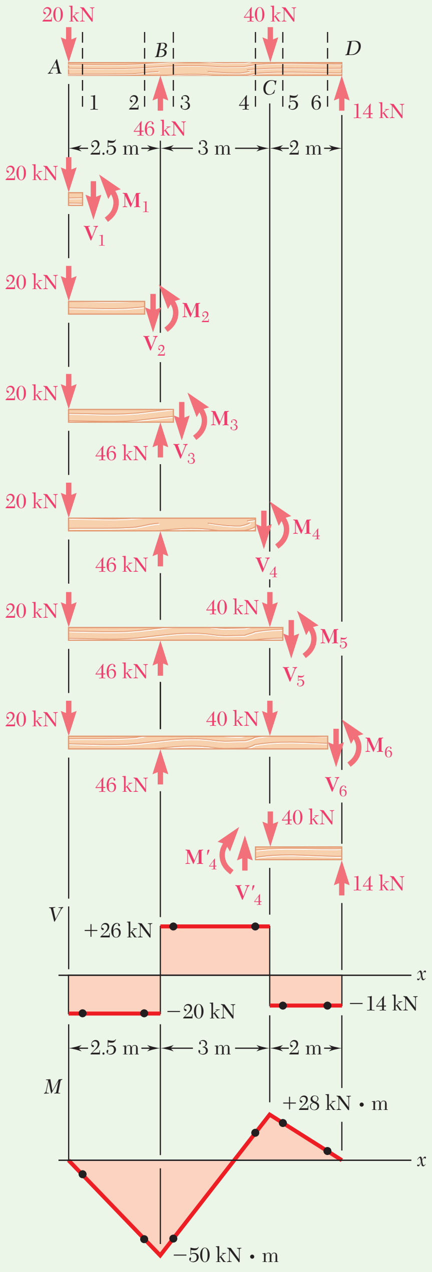


Fig. 1 Free-body diagram of beam, free-body diagrams of sections to left of cut, shear diagram, bending-moment diagram.

STRATEGY: After using statics to find the reaction forces, identify sections to be analyzed. You should section the beam at points to the immediate left and right of each concentrated force to determine values of V and M at these points.

MODELING and ANALYSIS:

Reactions. Considering the entire beam to be a free body (Fig. 1),

$$\mathbf{R}_B = 40\text{ kN}\uparrow \qquad \mathbf{R}_D = 14\text{ kN}\uparrow$$

Shear and Bending-Moment Diagrams. Determine the internal forces just to the right of the 20-kN load at A. Considering the stub of beam to the left of section *1* as a free body and assuming V and M to be positive (according to the standard convention), write

$$+\uparrow\Sigma F_y = 0: \qquad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\text{ kN}$$

$$+\circlearrowleft\Sigma M_1 = 0: \qquad (20\text{ kN})(0\text{ m}) + M_1 = 0 \qquad M_1 = 0$$

Next consider the portion to the left of section *2* to be a free body and write

$$+\uparrow\Sigma F_y = 0: \qquad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\text{ kN}$$

$$+\circlearrowleft\Sigma M_2 = 0: \qquad (20\text{ kN})(2.5\text{ m}) + M_2 = 0 \qquad M_2 = -50\text{ kN}\cdot\text{m}$$

The shear and bending moment at sections *3, 4, 5,* and *6* are determined in a similar way from the free-body diagrams shown in Fig. 1:

$$V_3 = +26\text{ kN} \qquad M_3 = -50\text{ kN}\cdot\text{m}$$

$$V_4 = +26\text{ kN} \qquad M_4 = +28\text{ kN}\cdot\text{m}$$

$$V_5 = -14\text{ kN} \qquad M_5 = +28\text{ kN}\cdot\text{m}$$

$$V_6 = -14\text{ kN} \qquad M_6 = 0$$

(continued)

For several of the latter sections, the results may be obtained more easily by considering the portion to the right of the section to be a free body. For example, for the portion of beam to the right of section 4,

$$+\uparrow \Sigma F_y = 0: \qquad V_4 - 40\text{ kN} + 14\text{ kN} = 0 \qquad V_4 = +26\text{ kN}$$

$$+\circlearrowleft \Sigma M_4 = 0: \qquad -M_4 + (14\text{ kN})(2\text{ m}) = 0 \qquad M_4 = +28\text{ kN}\cdot\text{m}$$

Now plot the six points shown on the shear and bending-moment diagrams. As indicated earlier, the shear is of constant value between concentrated loads, and the bending moment varies linearly.

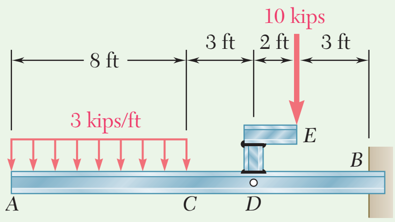
Maximum Normal Stress. This occurs at B, where $|M|$ is largest. Use Eq. (5.4) to determine the section modulus of the beam:

$$S = \tfrac{1}{6}bh^2 = \tfrac{1}{6}(0.080\text{ m})(0.250\text{ m})^2 = 833.33 \times 10^{-6}\text{ m}^3$$

Substituting this value and $|M| = |M_B| = 50 \times 10^3$ N·m into Eq. (5.3) gives

$$\sigma_m = \frac{|M_B|}{S} = \frac{(50 \times 10^3\text{ N}\cdot\text{m})}{833.33 \times 10^{-6}} = 60.00 \times 10^6\text{ Pa}$$

Maximum normal stress in the beam **= 60.0 MPa** ◀



Sample Problem 5.2

The structure shown consists of a W10 × 112 rolled-steel beam AB and two short members welded together and to the beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) Determine the maximum normal stress in sections just to the left and just to the right of point D.

STRATEGY: You should first replace the 10-kip load with an equivalent force-couple system at D. You can section the beam within each region of continuous load (including regions of no load) and find equations for the shear and bending moment.

MODELING and ANALYSIS:

Equivalent Loading of Beam. The 10-kip load is replaced by an equivalent force-couple system at D. The reaction at B is determined by considering the beam to be free body (Fig. 1).

(continued)

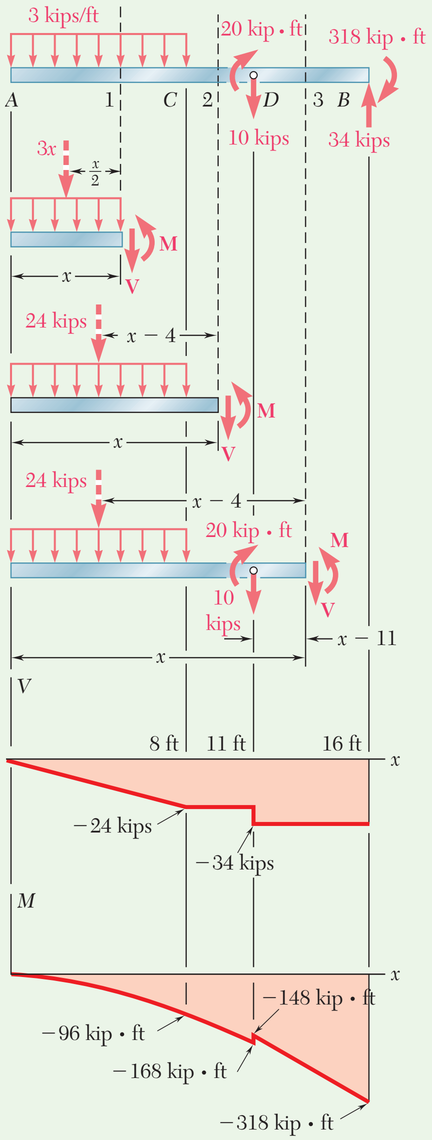


Fig. 1 Free-body diagram of beam, free-body diagrams of sections to left of cut, shear diagram, bending-moment diagram.

a. Shear and Bending-Moment Diagrams

From A to C. Determine the internal forces at a distance x from point A by considering the portion of beam to the left of section *1*. That part of the distributed load acting on the free body is replaced by its resultant, and

$$+\uparrow \Sigma F_y = 0: \qquad -3x - V = 0 \qquad V = -3x \text{ kips}$$
$$+\circlearrowleft \Sigma M_1 = 0: \qquad 3x(\tfrac{1}{2}x) + M = 0 \qquad M = -1.5x^2 \text{ kip·ft}$$

Since the free-body diagram shown in Fig. 1 can be used for all values of x smaller than 8 ft, the expressions obtained for V and M are valid in the region $0 < x < 8$ ft.

From C to D. Considering the portion of beam to the left of section *2* and again replacing the distributed load by its resultant,

$$+\uparrow \Sigma F_y = 0: \qquad -24 - V = 0 \qquad V = -24 \text{ kips}$$
$$+\circlearrowleft \Sigma M_2 = 0: \qquad 24(x - 4) + M = 0 \qquad M = 96 - 24x \quad \text{kip·ft}$$

These expressions are valid in the region 8 ft $< x <$ 11 ft.

From D to B. Using the position of beam to the left of section *3*, the region 11 ft $< x <$ 16 ft is

$$V = -34 \text{ kips} \qquad M = 226 - 34x \quad \text{kip·ft}$$

The shear and bending-moment diagrams for the entire beam now can be plotted. Note that the couple of moment 20 kip·ft applied at point D introduces a discontinuity into the bending-moment diagram.

b. Maximum Normal Stress to the Left and Right of Point *D*.

From Appendix C for the W10 × 112 rolled-steel shape, $S = 126 \text{ in}^3$ about the *X-X* axis.

To the left of D: $|M| = 168$ kip·ft $= 2016$ kip·in. Substituting for $|M|$ and S into Eq. (5.3), write

$$\sigma_m = \frac{|M|}{S} = \frac{2016 \text{ kip·in.}}{126 \text{ in}^3} = 16.00 \text{ ksi} \qquad \boldsymbol{\sigma_m = 16.00 \text{ ksi}} \blacktriangleleft$$

To the right of D: $|M| = 148$ kip·ft $= 1776$ kip·in. Substituting for $|M|$ and S into Eq. (5.3), write

$$\sigma_m = \frac{|M|}{S} = \frac{1776 \text{ kip·in.}}{126 \text{ in}^3} = 14.10 \text{ ksi} \qquad \boldsymbol{\sigma_m = 14.10 \text{ ksi}} \blacktriangleleft$$

REFLECT and THINK: It was not necessary to determine the reactions at the right end to draw the shear and bending-moment diagrams. However, having determined these at the start of the solution, they can be used as checks of the values at the right end of the shear and bending-moment diagrams.

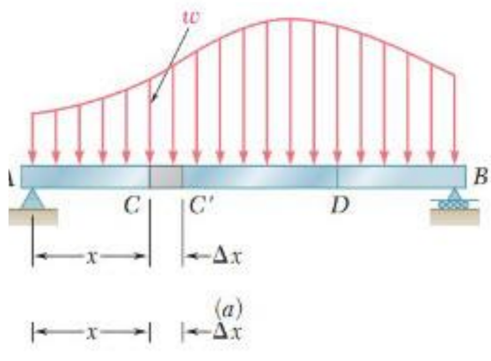
+ ارتباط بین بار، برش، گشتاور خمشی

Relationships between load, shear, and bending moment

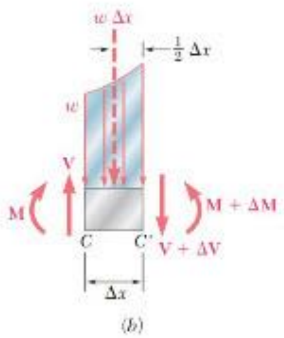
آنچه نشان داده شد تاکید می‌کند که روش پیشنهاد شده برای رسم دیاگرام‌های برش، گشتاور خمشی در صورتیکه تعداد بارهای متمرکز ۲ و بیشتر باشد یا بار گسترده بر تیر اعمال شده باشد، مشکل و زمان بر است.

بنابراین اگر ارتباطی بین بار، برش و گشتاور خمشی وجود داشته باشد رسم نمودارهای فوق را آسانتر می‌کند.

با در نظر گرفتن شکل‌های زیر، رابطه‌های فوق به این صورت بیان می‌شود:



(a)



(b)

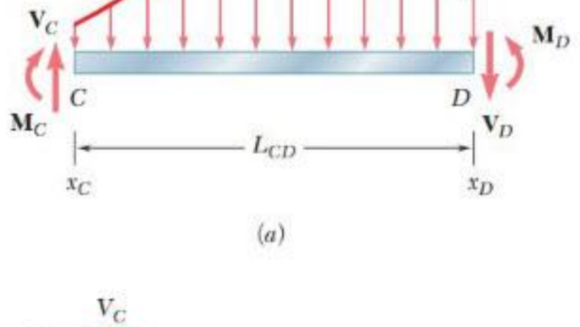
الف) رابطه بین بار گسترده و نیروی برشی

$$\frac{dV}{dx} = -w$$

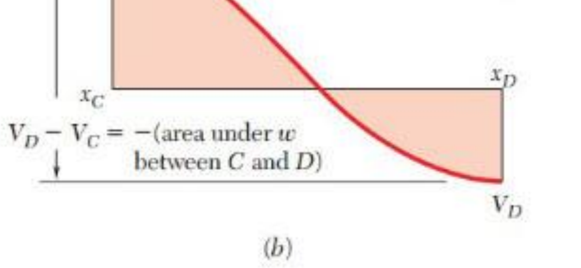
$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx = -(\text{ناحیه زیر منحنی بار در بازه } CD)$$

ب) رابطه بین برش و گشتاور خمشی

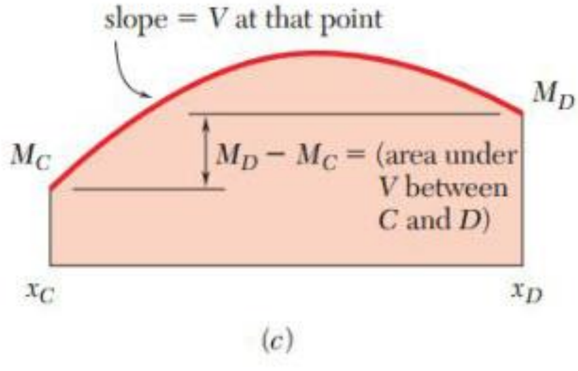
$$\begin{cases} \dfrac{dM}{dx} = V \\ M_D - M_C = \displaystyle\int_{x_C}^{x_D} V\,dx = \text{ناحیه زیر منحنی نیروی برشی در بازه } CD \end{cases}$$



(a)



(b)



(c)

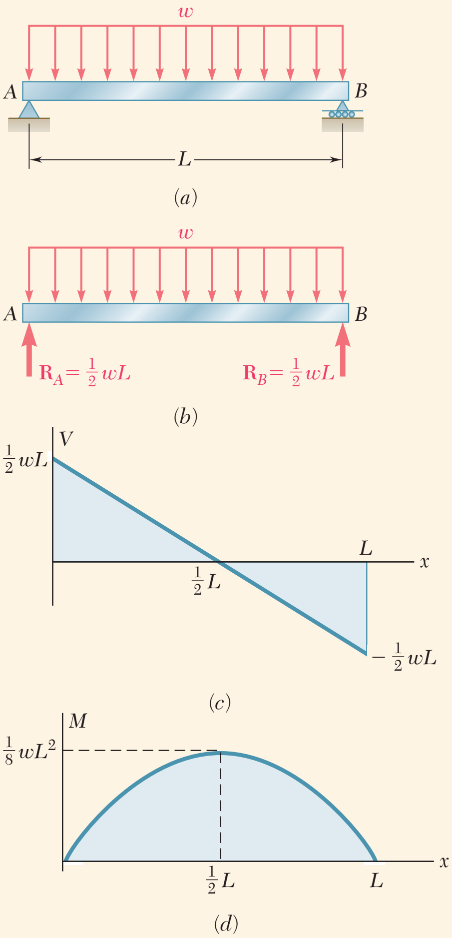


Fig. 5.11 *(a)* Simply supported beam with uniformly distributed load. *(b)* Free-body diagram. *(c)* Shear diagram. *(d)* Bending-moment diagram.

Concept Application 5.3

Draw the shear and bending-moment diagrams for the simply supported beam shown in Fig. 5.11*a* and determine the maximum value of the bending moment.

From the free-body diagram of the entire beam (Fig. 5.11*b*), we determine the magnitude of the reactions at the supports:

$$R_A = R_B = \tfrac{1}{2}wL$$

Next, draw the shear diagram. Close to the end A of the beam, the shear is equal to R_A, (that is, to $\frac{1}{2}wL$) which can be checked by considering as a free body a very small portion of the beam. Using Eq. (5.6*a*), the shear V at any distance x from A is

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \tfrac{1}{2}wL - wx = w(\tfrac{1}{2}L - x)$$

Thus the shear curve is an oblique straight line that crosses the x axis at $x = L/2$ (Fig. 5.11*c*). Considering the bending moment, observe that $M_A = 0$. The value M of the bending moment at any distance x from A is obtained from Eq. (5.8*a*):

$$M - M_A = \int_0^x V\,dx$$

$$M = \int_0^x w(\tfrac{1}{2}L - x)dx = \tfrac{1}{2}w(Lx - x^2)$$

The bending-moment curve is a parabola. The maximum value of the bending moment occurs when $x = L/2$, since V (and thus dM/dx) is zero for this value of x. Substituting $x = L/2$ in the last equation, $M_{max} = wL^2/8$ (Fig. 5.11*d*).

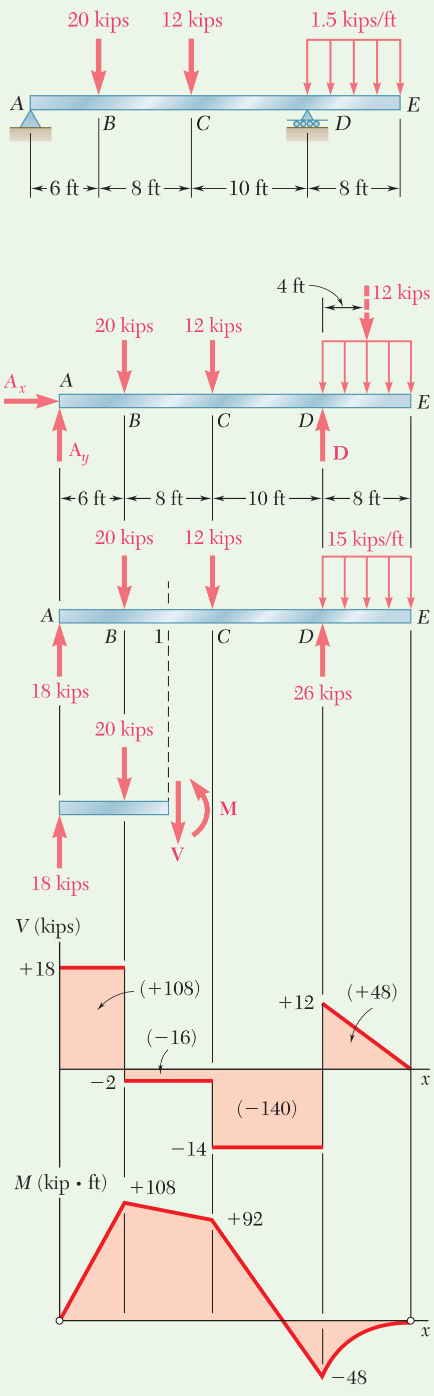


Fig. 1 Free-body diagrams of beam, free-body diagram of section to left of cut, shear diagram, bending-moment diagram.

Sample Problem 5.3

Draw the shear and bending-moment diagrams for the beam and loading shown.

STRATEGY: The beam supports two concentrated loads and one distributed load. You can use the equations in this section between these loads and under the distributed load, but you should expect changes in the diagrams at the concentrated load points.

MODELING and ANALYSIS:

Reactions. Consider the entire beam as a free body as shown in Fig. 1.

$+\circlearrowleft \Sigma M_A = 0$:

$$D(24\text{ ft}) - (20\text{ kips})(6\text{ ft}) - (12\text{ kips})(14\text{ ft}) - (12\text{ kips})(28\text{ ft}) = 0$$

$$D = +26\text{ kips} \qquad \mathbf{D} = 26\text{ kips}\uparrow$$

$+\uparrow\Sigma F_y = 0$: $\quad A_y - 20\text{ kips} - 12\text{ kips} + 26\text{ kips} - 12\text{ kips} = 0$

$$A_y = +18\text{ kips} \qquad \mathbf{A}_y = 18\text{ kips}\uparrow$$

$\xrightarrow{+}\Sigma F_x = 0$: $\quad A_x = 0 \qquad \mathbf{A}_x = 0$

Note that at both A and E the bending moment is zero. Thus, two points (indicated by dots) are obtained on the bending-moment diagram.

Shear Diagram. Since $dV/dx = -w$, between concentrated loads and reactions the slope of the shear diagram is zero (i.e., the shear is constant). The shear at any point is determined by dividing the beam into two parts and considering either part to be a free body. For example, using the portion of beam to the left of section *1*, the shear between B and C is

$$+\uparrow\Sigma F_y = 0: \quad +18\text{ kips} - 20\text{ kips} - V = 0 \qquad V = -2\text{ kips}$$

Also, the shear is $+12$ kips just to the right of D and zero at end E. Since the slope $dV/dx = -w$ is constant between D and E, the shear diagram between these two points is a straight line.

Bending-Moment Diagram. Recall that the area under the shear curve between two points is equal to the change in bending moment between the same two points. For convenience, the area of each portion of the shear diagram is computed and indicated in parentheses on the diagram in Fig. 1. Since the bending moment M_A at the left end is known to be zero,

$$M_B - M_A = +108 \qquad M_B = +108\text{ kip}\cdot\text{ft}$$
$$M_C - M_B = -16 \qquad M_C = +92\text{ kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -48\text{ kip}\cdot\text{ft}$$
$$M_E - M_D = +48 \qquad M_E = 0$$

Since M_E is known to be zero, a check of the computations is obtained.

(continued)

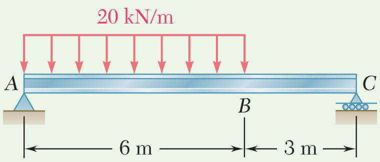
Between the concentrated loads and reactions, the shear is constant. Thus, the slope dM/dx is constant, and the bending-moment diagram is drawn by connecting the known points with straight lines. Between D and E where the shear diagram is an oblique straight line, the bending-moment diagram is a parabola.

From the V and M diagrams, note that $V_{max} = 18$ kips and $M_{max} = 108$ kip · ft.

REFLECT and THINK: As expected, the shear and bending-moment diagrams show abrupt changes at the points where the concentrated loads act.

Sample Problem 5.4

The W360 × 79 rolled-steel beam AC is simply supported and carries the uniformly distributed load shown. Draw the shear and bending-moment diagrams for the beam, and determine the location and magnitude of the maximum normal stress due to bending.



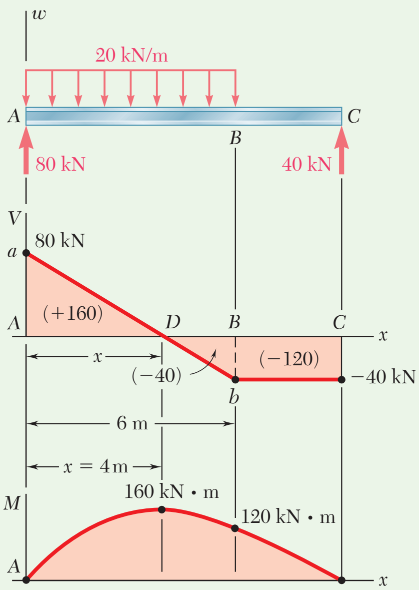


Fig. 1 Free-body diagram, shear diagram, bending-moment diagram.

STRATEGY: A load is distributed over part of the beam. You can use the equations in this section in two parts: for the load and for the no-load regions. From the discussion in this section, you can expect the shear diagram will show an oblique line under the load, followed by a horizontal line. The bending-moment diagram should show a parabola under the load and an oblique line under the rest of the beam.

MODELING and ANALYSIS:

Reactions. Considering the entire beam as a free body (Fig. 1),

$$\mathbf{R}_A = 80\text{ kN} \uparrow \qquad \mathbf{R}_C = 40\text{ kN} \uparrow$$

Shear Diagram. The shear just to the right of A is $V_A = +80$ kN. Since the change in shear between two points is equal to *minus* the area under the load curve between the same two points, V_B is

$$V_B - V_A = -(20\text{ kN/m})(6\text{ m}) = -120\text{ kN}$$
$$V_B = -120 + V_A = -120 + 80 = -40\text{ kN}$$

(continued)

The slope $dV/dx = -w$ is constant between A and B, and the shear diagram between these two points is represented by a straight line. Between B and C, the area under the load curve is zero; therefore,

$$V_C - V_B = 0 \qquad V_C = V_B = -40 \text{ kN}$$

and the shear is constant between B and C.

Bending-Moment Diagram. Note that the bending moment at each end is zero. In order to determine the maximum bending moment, locate the section D of the beam where $V = 0$.

$$V_D - V_A = -wx$$

$$0 - 80 \text{ kN} = -(20 \text{ kN/m})x$$

Solving for x, **$x = 4$ m** ◀

The maximum bending moment occurs at point D, where $dM/dx = V = 0$. The areas of various portions of the shear diagram are computed and given (in parentheses). The area of the shear diagram between two points is equal to the change in bending moment between the same two points, giving

$$M_D - M_A = +160 \text{ kN·m} \qquad M_D = +160 \text{ kN·m}$$

$$M_B - M_D = -40 \text{ kN·m} \qquad M_B = +120 \text{ kN·m}$$

$$M_C - M_B = -120 \text{ kN·m} \qquad M_C = 0$$

The bending-moment diagram consists of an arc of parabola followed by a segment of straight line. The slope of the parabola at A is equal to the value of V at that point.

Maximum Normal Stress. This occurs at D, where $|M|$ is largest. From Appendix C, for a W360 × 79 rolled-steel shape, $S = 1270 \text{ mm}^3$ about a horizontal axis. Substituting this and $|M| = |M_D| = 160 \times 10^3$ N·m into Eq. (5.3),

$$\sigma_m = \frac{|M_D|}{S} = \frac{160 \times 10^3 \text{ N·m}}{1270 \times 10^{-6} \text{ m}^3} = 126.0 \times 10^6 \text{ Pa}$$

Maximum normal stress in the beam = **126.0 MPa** ◀

Sample Problem 5.5

Sketch the shear and bending-moment diagrams for the cantilever beam shown in Fig. 1.

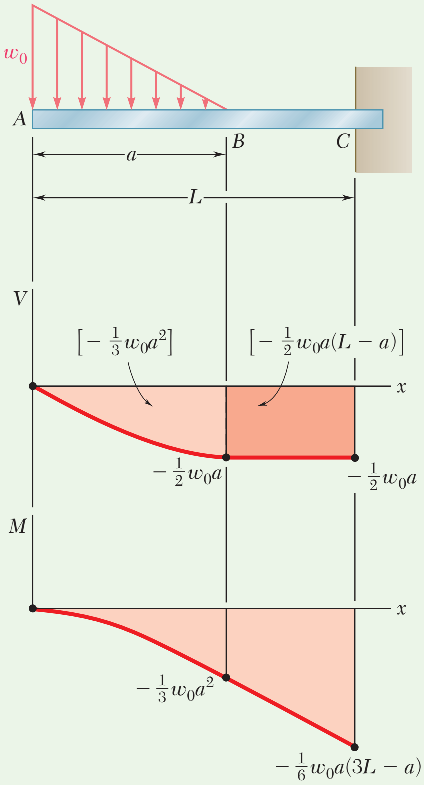


Fig. 1 Beam with load, shear diagram, bending-moment diagram.

STRATEGY: Because there are no support reactions until the right end of the beam, you can rely solely on the equations from this section without needing to use free-body diagrams and equilibrium equations. Due to the non-uniform distributed load, you should expect the results to involve equations of higher degree, with a parabolic curve in the shear diagram and a cubic curve in the bending-moment diagram.

MODELING and ANALYSIS:

Shear Diagram. At the free end of the beam, $V_A = 0$. Between A and B, the area under the load curve is $\frac{1}{2}w_0a$. Thus,

$$V_B - V_A = -\tfrac{1}{2}w_0a \qquad V_B = -\tfrac{1}{2}w_0a$$

Between B and C, the beam is not loaded, so $V_C = V_B$. At A, $w = w_0$. According to Eq. (5.5), the slope of the shear curve is $dV/dx = -w_0$, while at B the slope is $dV/dx = 0$. Between A and B, the loading decreases linearly, and the shear diagram is parabolic. Between B and C, $w = 0$, and the shear diagram is a horizontal line.

Bending-Moment Diagram. The bending moment M_A at the free end of the beam is zero. Compute the area under the shear curve to obtain.

$$M_B - M_A = -\tfrac{1}{3}w_0a^2 \qquad M_B = -\tfrac{1}{3}w_0a^2$$

$$M_C - M_B = -\tfrac{1}{2}w_0a(L-a)$$

$$M_C = -\tfrac{1}{6}w_0a(3L-a)$$

The sketch of the bending-moment diagram is completed by recalling that $dM/dx = V$. Between A and B, the diagram is represented by a cubic curve with zero slope at A and between B and C by a straight line.

REFLECT and THINK: Although not strictly required for the solution of this problem, determination of the support reactions would serve as an excellent check of the final values of the shear and bending-moment diagrams.

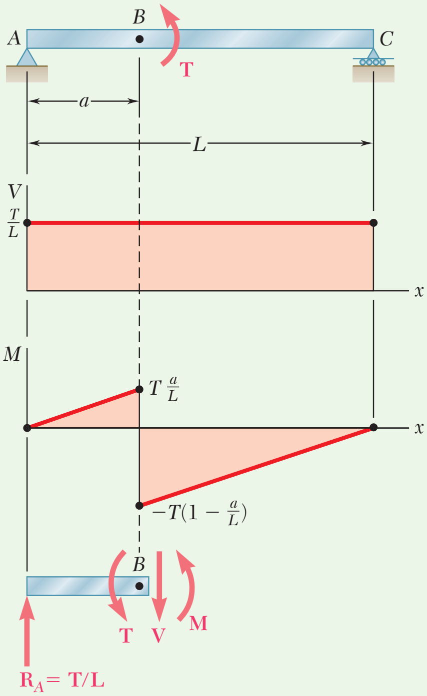


Fig. 1 Beam with load, shear diagram, bending-moment diagram, free-body diagram of section to left of *B*.

Sample Problem 5.6

The simple beam *AC* in Fig. 1 is loaded by a couple of moment *T* applied at point *B*. Draw the shear and bending-moment diagrams of the beam.

STRATEGY: The load supported by the beam is a concentrated couple. Since the only vertical forces are those associated with the support reactions, you should expect the shear diagram to be of constant value. However, the bending-moment diagram will have a discontinuity at *B* due to the couple.

MODELING and ANALYSIS:

The entire beam is taken as a free body.

$$\mathbf{R}_A = \frac{T}{L}\uparrow \qquad \mathbf{R}_C = \frac{T}{L}\downarrow$$

The shear at any section is constant and equal to T/L. Since a couple is applied at *B*, the bending-moment diagram is discontinuous at *B*. It is represented by two oblique straight lines and decreases suddenly at *B* by an amount equal to *T*. This discontinuity can be verified by equilibrium analysis. For example, considering the free body of the portion of the beam from *A* to just beyond the right of *B* as shown in Fig. 1, *M* is

$$+\circlearrowleft\Sigma M_B = 0: \quad -\frac{T}{L}a + T + M = 0 \qquad M = -T\left(1 - \frac{a}{L}\right)$$

REFLECT and THINK: Notice that the applied couple results in a sudden change to the moment diagram at the point of application in the same way that a concentrated force results in a sudden change to the shear diagram.