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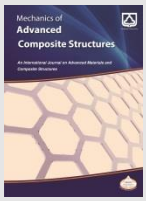

Nonlocal Analysis of Longitudinal Dynamic Behavior of Nanobars with Surface Energy Effect

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ABSTRACT

Due to considerable stored energy in surfaces of nano-scales in comparison with the stored energy in their bulk, considering the surface energy is necessary for the analysis of various behaviors of nano-scales for more precise design and manufacturing. In this article, the longitudinal dynamic behavior of nanobars in the presence of the surface energy parameters is studied. To this end, the longitudinal dynamic behavior of nanobars is modeled based on the simple theory. To consider the effects of the surface energy parameters, the surface elasticity theory is used. In addition, the nonlocal elasticity theory is implemented to capture the size effect. Then, the governing equation of motion and corresponding boundary conditions are derived from Hamilton's principle. The governing equation becomes the inhomogeneous cause of considering the surface energy parameters while in none of the previous researches like the investigation of transverse vibration of nanobeams and torsional vibration of nanobars, the surface energy parameters would not cause inhomogeneity of the governing equation. Due to inhomogeneity of the governing equation, the homogeneous case is firstly solved, and frequencies and mode shapes of nanobar are obtained for fixed-fixed and fixed-free boundary conditions. Then, using the modal analysis method and Duhamel's integral, the inhomogeneous governing equation of motion is solved, and the overall dynamic response of nanobar is reported.

1. Introduction

Science and technology have been continuously in progress, and human is looking for improvement in every field. Hence, in recent years, the researchers have been very interested to work on nanoscale materials and structures due to their unique mechanical, electrical, and physical properties. One of the problems facing nanoscale researchers is the difference in nanoscale properties and behaviors compared to similar systems in macro dimensions. The reason for this difference is due to the fact that various behaviors of nanoscales are size-dependent. To model the dependency of nanoscale properties on their dimensions, various theories such as surface elasticity theory, nonlocal elasticity theory, strain gradient theory, couple stress theory, and modified couple stress theory have been proposed. In theories of nonlocal elasticity, strain gradient, couple stress, and modified couple stress, by defining a parameter, called the small scale parameter, effects of dimensions on various behaviors of nanoscale structures would be considered. In the theory of surface elasticity, the energy of nanostructure surfaces, in addition to the energy generated by its volume would be considered in equations of motion and boundary conditions. The reason is that the atoms located at nanostructure surfaces are exposed to a different environment than those in bulk. This difference would result that the

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equilibrium positions and energies of surface atoms differ from the atoms in bulk.

Using the above theories, the mechanical behaviors of various nanoscale structures have been studied [1-3]. Among these studies, longitudinal vibration analysis of nanobars is less paid attention. However, there are some researches using the small scale parameter for longitudinal vibration of nanorods. Aydogdu [4, 5] studied the longitudinal free vibrations of nanorods and embedded nanorods in the elastic environment utilizing the nonlocal elasticity theory and investigated the effect of the nonlocal parameter on longitudinal frequencies. The free longitudinal vibrational behavior of nanorods with variable cross-sectional area taking into account the effect of nonlocal parameter had also been investigated [6-8]. Hsu et al. [9] considered the nonlocal effect on free longitudinal vibration of cracked nanorods. The nonlocal elasticity theory was used to analyze the free longitudinal vibrational behavior of two connected nanorods. The effect of Van der Waals interactions was also considered [10, 11]. In another study, the free longitudinal vibration of nanorods was analyzed by considering the effect of lateral displacement of nanorods modeled based on the nonlocal Rayleigh theory and the nonlocal elasticity theory [12]. Free axial vibration of thick functionally graded size-dependent nanorods was investiagted by Nazemnezhad and Kamali [13]. It is also worth to mention the study analyzing the free longitudinal vibration of nanorods using the strain gradient theory [14]. The other related works can be found in [15-19].

Researches done on analysis of the longitudinal or axial vibrational behavior of nanorods/nanobars show that they have used the nonlocal elasticity theory or strain gradient theory. Therefore, there is a question of how the longitudinal dynamic behavior of nanorods/nanobars is based on the theory of surface elasticity? However, there are some studies that consider various mechanical behaviors of nanostructures using the surface elasticity theory [20-25].

To cover this issue, in the present study, the longitudinal dynamic behavior of nanobars is investigated by considering the surface energy components. To this end, the longitudinal dynamic behavior of the nanobar is modeled based on the surface elasticity and classical elasticity theory, and the equation of motion and boundary conditions of the nanobar is extracted by using Hamilton's principle. In the next step, the equation of motion is solved analytically for two boundary conditions; i.e., fixed-fixed and fixed-free. Finally, in the results section, the influence of geometrical factors such as the length and radius of nanobar, along with the effect of surface energy components on the longitudinal dynamic behavior of the nanobar has been investigated. In addition, a comparison is made between the effects of the surface energy components on the various behaviors of nanostructures; i.e., longitudinal vibration, transverse vibration, and torsional vibration.

2. Problem Formulation

To extract the governing equation of motion for the free longitudinal vibrational motion of nanobars taking into account the surface energy effect, a nanobar with length L $\left(0 \le x \le L\right)$ and radius R is considered in the *x-y-z* coordinates system (Fig. 1).

The next step for problem formulation is selecting an appropriate theory for modeling the longitudinal vibrational behavior of nanobars. To this end, it is assumed that 1) the cross-section of the nanobar originally plane remain plane during deformation, and 2) the displacement components in the nanobar (except for the component parallel to the nanobar longitudinal axis) are negligible. These assumptions introduce the Simple theory of rods. The displacement field of any point of the nanobar based on the Simple theory of rods could be expressed as [26, 27]:

$$\begin{cases} u(x.\,\mathrm{y}.\,\mathrm{z}.\,t) = u_0(x.\,t); \\ v(x.\,\mathrm{y}.\,\mathrm{z}.\,t) = w(x.\,\mathrm{y}.\,\mathrm{z}.\,t) = 0 \end{cases} \tag{1}$$

in which u, v and w are displacement components of nanobar along the x, y and z directions, respectively, and t is the time. Based on the components of the displacement field, the strains in the cross-section of nanobar are given by:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x};\ \ \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{yy} = \varepsilon_{zz} = 0 \tag{2}$$

The stresses of the nanobar are also given by:

$$\begin{cases} \sigma_{xx} = E\dfrac{\partial u_0}{\partial x}; \\ \sigma_{xy} = \sigma_{xz} = \sigma_{yz} = \sigma_{yy} = \sigma_{zz} = 0 \end{cases} \tag{3}$$

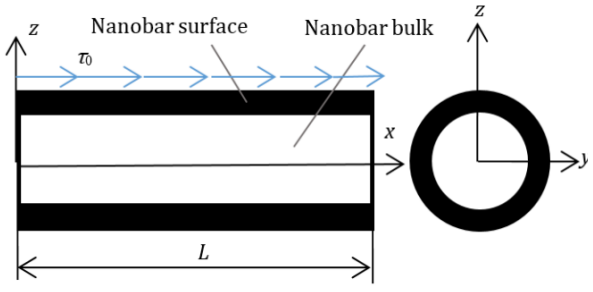

Fig. 1. Schematic of a nanobar with its surface.

Eqs. (2) and (3) are the strain and stress components of the nanobar volume, respectively. Since Hamilton's principle requires the strains and stresses of both the surface and volume of nanobar, the components of surface strain and surface stress are obtained.

The surface elasticity theory is implemented to calculate the stress components at the nanobar surface. In the surface elasticity theory of Gurtin and Murdak [28], the surface stresses are expressed as:

$$\begin{aligned}\tau_{\alpha\beta} &= \tau_0\ \delta_{\alpha\beta} + 2(\mu_0 - \tau_0)\varepsilon_{\alpha\beta} \\ &\quad + (\lambda_0 + \tau_0)u_{\gamma\gamma}\delta_{\alpha\beta} \\ &\quad + \tau_0\ u_{\alpha\beta}; \qquad (\alpha.\beta = x.y)\end{aligned} \tag{4}$$

$$\tau_{\alpha z} = \tau_0\ \frac{\partial u_0}{\partial \alpha}; \qquad (\alpha = x.y) \tag{5}$$

where λ_0 and μ_0 are the surface Lamé constants, $\delta_{\alpha\beta}$ is the Kronecker delta, and τ_0 is the surface residual stress.

Before using Eqs. (4) and (5), it is worth to note that Wang and Feng [29] have considered free transverse vibration of microbeams incorporating the surface energy effects. They have modeled the residual surface stress as an external work acting on the microbeam. Therefore, the one has done in Ref. [29] was selected for considering the effect of the residual surface stress. This yields:

$$W = \int_0^L \left(\tau_0 u_0(x.t)\right)dx \tag{6}$$

Substituting displacement components (Eq. (1)) into Eqs. (4) and (5) results in the surface stresses as:

$$\begin{cases}\tau_{xx} = (2\mu_0 + \lambda_0)\dfrac{\partial u_0}{\partial x} \\ \tau_{yy} = (\lambda_0)\dfrac{\partial u_0}{\partial x} \\ \tau_{xy} = \tau_{xz} = \tau_{yz} = \tau_{zz} = 0\end{cases} \tag{7}$$

Now, it is possible to form the potential energy of the nanobar as:

$$U = \int (\sigma_{xx}\varepsilon_{xx})dV + \int \tau_{xx}\varepsilon_{xx}dA \tag{8}$$

The next parameter in Hamilton's principle is the kinetic energy of the nanobar. This parameter is given by:

$$\begin{aligned}T = \frac{1}{2}\int \rho\left(\frac{\partial u_0(x.t)}{\partial t}\right)^2 dV \\ + \frac{1}{2}\int \rho_0\left(\frac{\partial u_0(x.t)}{\partial t}\right)^2 dA\end{aligned} \tag{9}$$

where ρ (kg/m^3) and ρ_0 (kg/m^2) are the density of the volume and surface of nanobar, respectively, V and A indicate the volume and surface of the nanobar and $dA = 2\,\pi R dx$. Now that the necessary components of Hamilton's principle (Eq. (10)) are obtained, the governing equation of motion and corresponding boundary conditions could be derived by putting Eqs. (6), (8) and (9) into Eq. (10), and performing the integral by parts. This would result in:

$$\delta\int_{t_1}^{t_2}(U - T - W)dt = 0 \tag{10}$$

$$\frac{\partial N_{xx}}{\partial x} - (\rho A)_{eq}\frac{\partial^2 u_0}{\partial t^2} = -\tau_0 \tag{11}$$

$$(N_{xx}\delta u_0)\Big|_0^l = 0 \tag{12}$$

where

$$N_{xx} = \int \sigma_{xx}dA + \int \tau_{xx}dS \tag{13}$$

$$\begin{cases}(\rho A)_{eq} = \rho A + \rho_0 S \\ A = \pi R^2 \\ S = 2\pi R\end{cases}; \tag{14}$$

Eqs. (11) and (12) are the local governing equation of motion and corresponding boundary condition for a nanobar in the free axial vibration. To obtain their nonlocal form, the nonlocal elasticity theory should be used [30]. Based on the theory:

$$\begin{aligned}(1 - \mu\nabla^2)N_{xx}^{nl} &= N_{xx} \\ &= \int \sigma_{xx}dA + \int \tau_{xx}dS \\ &= (EA + E_0 S)\frac{\partial u_0}{\partial x} \\ &= (EA)_{eq}\frac{\partial u_0}{\partial x}\end{aligned} \tag{15}$$

$$\begin{aligned}\left((1 - \mu\nabla^2)N_{xx}^{nl}\delta u_0\right)\Big|_0^l &= 0 \\ \Rightarrow \left((EA)_{eq}\frac{\partial u_0}{\partial x}\delta u_0\right)\Big|_0^l &= 0 \Rightarrow \left(\frac{\partial u_0}{\partial x}\delta u_0\right)\Big|_0^l = 0\end{aligned} \tag{16}$$

where μ is the nonlocal parameter, $\nabla^2 = \partial^2/\partial x^2$ is the one-dimensional Laplacian operator, and superscript *nl* denotes the nonlocal.

Multiplying both sides of Eq. (11) by $(1 - \mu\nabla^2)$ and substituting Eq. (15) into Eq. (11) would result in the nonlocal equation of motion in terms of deflection as follows:

$$c^2\frac{\partial^2 u_0}{\partial x^2} + \mu\frac{\partial^4 u_0}{\partial x^2\partial t^2} - \frac{\partial^2 u_0}{\partial t^2} = -\frac{\tau_0}{(\rho A)_{eq}} \tag{17}$$

where $c^2 = (EA)_{eq}/(\rho A)_{eq}$.

According to Eq. (16), the following boundary conditions could be considered:

- Fixed-Fixed $$u_0(0.t) = u_0(L.t) = 0 \quad (18)$$
- Fixed-Free $$u_0(0.t) = \frac{\partial u_0(L.t)}{\partial x} = 0 \quad (19)$$

2.1. Solution Procedure

Since the nonlocal governing equation of motion (Eq. (17)) is an inhomogeneous equation, firstly its general solution requires the solution of its homogeneous part. To this end, Eq. (17) is written as:

$$c^2 \frac{\partial^2 u_0}{\partial x^2} + \mu \frac{\partial^4 u_0}{\partial x^2 \partial t^2} - \frac{\partial^2 u_0}{\partial t^2} = 0 \quad (20)$$

Eq. (20) indicates the free longitudinal vibrational behavior of nanobars incorporating the surface energy effects. To solve Eq. (20), a harmonic displacement for the nanobar is assumed as:

$$u_0(x.t) = U(x)e^{i\omega t} \quad (21)$$

in which ω is the natural longitudinal frequency. Substituting Eq. (21) into Eqs. (18)-(20) results in the following equations:

$$c^2 \frac{d^2U(x)}{dx^2} - \mu\omega^2 \frac{d^2U(x)}{dx^2} + \omega^2 U(x) = 0 \quad (22)$$

- Fixed-Fixed $$U(0) = U(L) = 0 \quad (23)$$
- Fixed-Free $$U(0) = \frac{dU(L)}{dx} = 0 \quad (24)$$

The general solution of the second-order differential equation (Eq. (22)) is given by:

$$U(x) = A_1 \cos(\lambda x) + A_2 \sin(\lambda x) \quad (25)$$

where $\lambda = \sqrt{\omega^2/(c^2 - \mu\omega^2)}$.

To obtain the mode shapes and natural frequencies associated with each boundary condition type, Eqs. (23) and (24) should be applied to Eq. (25). This gives:

❖ Mode Shapes:

- Fixed-Fixed $$U_i(x) = A_i \sin\left(\frac{i\pi}{L}x\right) \quad (26)$$
- Fixed-Free $$U_i(x) = A_i \sin\left(\frac{(2i-1)\pi}{2L}x\right) \quad (27)$$

❖ Natural Frequencies:

- Fixed-Fixed $$\omega_i = \frac{\frac{i\pi c}{L}}{\sqrt{1 + \mu\left(\frac{i\pi}{L}\right)^2}} \quad (28)$$
- Fixed-Free $$\omega_i = \frac{\frac{(2i-1)\pi c}{2L}}{\sqrt{1 + \mu\left(\frac{(2i-1)\pi}{2L}\right)^2}} \quad (29)$$

Note that $i = 1, 2, 3, \ldots$.

After determining the nanobar mode shapes and natural frequencies, it is possible to solve the inhomogeneous part of the governing equation of motion (Eq. (11)). For this purpose, the modal analysis method, as well as the orthogonality relation for mode shapes, are used. The general response of nanobar during longitudinal vibrations could be considered as:

$$u_0(x.t) = \sum_{i=1}^{\infty} U_i(x)\eta_i(t) \quad (30)$$

where $U_i(x)$ is the nanobar mode shape, and $\eta_i(t)$ is the generalized coordinate. Substituting Eq. (30) into Eq. (17), multiplying the obtained equation by $U_j(x)$, and integrating over the length of the nanobar would yield:

$$\begin{aligned} &\sum_{i=1}^{\infty}\Bigg[c^2\eta_i(t)\int_0^L \frac{d^2U_i(x)}{dx^2}U_j(x)dx \\ &+ \ddot{\eta}_i(t)\mu\int_0^L \frac{d^2U_i(x)}{dx^2}U_j(x)dx \\ &- \ddot{\eta}_i(t)\int_0^L U_i(x)U_j(x)dx\Bigg] \\ &= -\frac{\tau_0}{(\rho A)_{eq}}\int_0^L U_j(x)dx \end{aligned} \quad (31)$$

Eq. (31) could be simplified using the orthogonality relation for the mode shapes. The orthogonality relations (which are valid not only among the mode shapes but also among the derivatives of the mode shapes) are given by [26]:

$$\int_0^L U_i(x)U_j(x)dx = \begin{cases} 0 & for\ i \neq j \\ 1 & for\ i = j \end{cases} \quad (32)$$

$$\int_0^L \frac{dU_i(x)}{dx}\frac{dU_j(x)}{dx}dx = \begin{cases} 0 & for\ i \neq j \\ -\left(\frac{\omega_i}{c}\right)^2 & for\ i = j \end{cases} \quad (33)$$

$$\int_0^L \frac{d^2U_i(x)}{dx^2}U_j(x)dx = \begin{cases} 0 & for\ i \neq j \\ -\left(\frac{\omega_i}{c}\right)^2 & for\ i = j \end{cases} \quad (34)$$

For *i=j*, the substitution of Eqs. (32) and (34) into Eq. (31) results in:

$$\ddot{\eta}_i(t) + B_1^2 \eta_i(t) = B_2 \int_0^L U_i(x)dx \tag{35}$$

where:

$$B_1 = \frac{c\omega_i}{\sqrt{c^2 + \mu\omega_i^2}}$$
$$B_2 = \frac{c^2\tau_0}{(\rho A)_{eq}(c^2 + \mu\omega_i^2)} \tag{36}$$

Eq. (35) is a second-order ordinary differential equation with respect to time. The solution of Eq. (35) using Duhamel's integral [31] is given by:

$$\eta_i(t) = \frac{1}{B_1}\int_0^L U_i(x)\left[\int_0^t B_2\, sin\big(B_1(t-\tau)\big)d\tau\right]dx \tag{37}$$

By obtaining a relation for generalized coordinates (Eq. (37)), the general response of nanobar, regardless of the initial conditions, is given by (substitute Eq. (37) into Eq. (30)):

$$u_0(x.t) = \sum_{i=1}^{\infty}\frac{U_i(x)}{B_1}\int_0^L U_i(x)\times \left[\int_0^t B_2\sin\big(B_1(t-\tau)\big)\,d\tau\right]dx \tag{38}$$

It is necessary to point out that in the use of Eq. (38), the coefficients of the mode shapes (see Eqs. (26)-(27)) must be obtained from the orthogonality conditions (Eqs. (32)-(34)). This yields:

- Fixed-Fixed $$U_i(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{i\pi}{L}x\right) \tag{39}$$

- Fixed-Free $$U_i(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{(2i-1)\pi}{2L}x\right) \tag{40}$$

Now, by substituting Eqs. (28), (29), (39) and (40) into Eq. (38), the general dynamic response of the nanobar for fixed-fixed and fixed-free boundary conditions is given by Eqs. (41) and (42), respectively.

$$u_0(x.t) = \sum_{i=1.3....}^{\infty}\left\{\frac{4\tau_0(L^2+\mu(i\pi)^2)}{(EA)_{eq}(i\pi)^3}\sin\left(\frac{i\pi}{L}x\right)\times(1-\cos(B_1t))\right\} \tag{41}$$

$$u_0(x.t) = \sum_{i=1}^{\infty}\frac{4\tau_0\left(4L^2+\mu\big((2i-1)\pi\big)^2\right)}{(EA)_{eq}\big((2i-1)\pi\big)^3}\sin\left(\frac{(2i-1)\pi}{2L}x\right)\times\left(1-\cos\left(\frac{(2i-1)\pi c}{2L}t\right)\right) \tag{42}$$

3. Results and Discussion

To verify the accuracy and exactitude of derived equations and solving method, the results of the present study are compared with those reported in the literature. Since no research has ever been found to examine the effects of the surface energy on the longitudinal vibration of nanorods, the present results are compared with those of Refs. [26] and [8]. In Ref. [26], local axial vibration, and in Ref. [8], nonlocal axial vibration of nanobars are studied. The modeling of nanobars is done based on the simple theory of rods. The comparison is listed in Tables 1 and 2. The results reported in Tables 1 and 2 show that the results of the present study are consistent with the other ones, which indicates the accuracy of the derived equations and the solving method.

Firstly, Eq. (38) is defined to present new results and investigate the effects of the surface energy parameters and the nonlocal parameter on the longitudinal dynamic behavior of nanobars.

$$\text{FR} = \frac{\text{Frequency with surface energy and nonlocal effects}}{\text{Frequency without surface energy and nonlocal effects}} \tag{38}$$

Then, two categories of results are reported. The results of the first category are concluded from the mathematical relations of natural frequencies (Eqs. (28)-(29)), mode shapes (Eqs. (26)-(27)), and dynamic responses (Eqs. (41)-(42)), and the results of the second category are numerical results that will be presented as tables and figures.

The results concluded from the mathematical relations of natural frequencies, mode shapes, and dynamic responses could be summarized as:

- Eq. (17) shows that considering the surface energy effects causes the inhomogeneity of the nanobar longitudinal governing equation of motion while this is not the case in researches like the investigation of the transverse vibration of nanobeams [32-35] and the investigation of the torsional vibration of nanorods [36, 37] incorporating the surface energy effects.
- Eqs. (26) and (27) show that the surface energy components do not have any effect on the longitudinal mode shapes of the nanobar. A literature survey shows that the same result has been reported for the effect of the surface energy components on torsional mode shapes of nanorods [37], but a different result has been observed for the effect of the surface energy components on the transverse vibration of nanobeams [38]. In free transverse vibration of nanobeams, from the three types of boundary condition, fixed-fixed, fixed-free and hinged-hinged, the surface

energy components affect only the mode shapes of the nanobeams with fixed-free boundary condition.

- Eqs. (28) and (29) show that the surface energy and the nonlocal would affect the natural longitudinal frequencies of nanobars with both fixed-fixed and fixed-free boundary conditions.
- According to the definition of the parameter c (see Eq. (17)) and Eqs. (28)-(29), it could be concluded that only the surface Lamé constants and the surface density would affect the longitudinal frequencies of nanobars. The surface stress does not affect the natural longitudinal frequencies. However, it has been reported in references that all surface energy components affect the natural torsional frequencies of nanorods [37] and the natural transverse frequencies of nanobeams [34].
- Eqs. (36) and (37) show that all surface energy components and the nonlocal parameter would affect the longitudinal dynamic response of the nanobar.

In the second category of results, the numerical results are presented as tables and figures. These results include variations of the natural longitudinal frequencies and frequency ratios versus the frequency number; variations of the natural longitudinal frequencies and frequency ratios versus the length of nanobar; and variations of the natural longitudinal frequencies and frequency ratios versus the radius of nanobar.

The numerical results are for nanobar with radius R, length L, and made of aluminum and silicon with the mechanical properties given in Table 3. The mechanical properties of aluminum are in [111] crystalline direction and silicon in [100] crystalline direction [34].

Table 1. Comparison of natural frequencies of nanobar for various values of its length (E=70 GPa, ρ=2700 kg/m³)

L (nm)	Mode number	Frequency (THz)					
		Fixed-Fixed			Fixed-Free		
		Present work	Ref. [8]	Ref. [26]	Present work	Ref. [8]	Ref. [26]
10	1	1.60	1.60	1.60	0.80	0.80	0.80
	2	3.20	3.20	3.20	2.40	2.40	2.40
	3	4.80	4.80	4.80	4.00	4.00	4.00
	4	6.40	6.40	6.40	5.60	5.60	5.60
	5	8.00	8.00	8.00	7.20	7.20	7.20
20	1	0.80	0.80	0.80	0.40	0.40	0.40
	2	1.60	1.60	1.60	1.20	1.20	1.20
	3	2.40	2.40	2.40	2.00	2.00	2.00
	4	3.20	3.20	3.20	2.80	2.80	2.80
	5	4.00	4.00	4.00	3.60	3.60	3.60
30	1	0.53	0.53	0.53	0.27	0.27	0.27
	2	1.07	1.07	1.07	0.80	0.80	0.80
	3	1.60	1.60	1.60	1.33	1.33	1.33
	4	2.13	2.13	2.13	1.87	1.87	1.87
	5	2.67	2.67	2.67	2.40	2.40	2.40

Table 2. Comparison of natural frequencies of nanobar for various values of the nonlocal parameter (E=70 GPa, ρ=2700 kg/m³)

μ (nm^2)	Mode number	Frequency (THz)			
		Fixed-Fixed		Fixed-Free	
		Present work	Ref. [8]	Present work	Ref. [8]
0.0025	1	1.58	1.58	0.8	0.8
	2	3.05	3.05	2.34	2.34
	3	4.34	4.34	3.72	3.72
	4	5.42	5.42	4.91	4.91
	5	6.29	6.29	5.88	5.88
0.0100	1	1.53	1.53	0.79	0.79
	2	2.71	2.71	2.17	2.17
	3	3.49	3.49	3.15	3.15
	4	3.98	3.98	3.77	3.77
	5	4.3	4.3	4.16	4.16

Table 3. Mechanical properties of aluminum and silicon nanobars.

Mechanical properties	aluminum	silicon
Volume Elasticity Modulus (GPa)	70	210
Surface Lamé constants $(2\mu_0 + \lambda_0)$ (N/m)	5.1882	-10.6543
Volume Density (kg/m^3)	2700	2370
Surface Density (kg/m^2)	5.46×10^{-7}	3.17×10^{-7}
Surface Stress (N/m)	0.9108	0.6048

In Table 4, values of the first four classic frequencies and frequency ratios of nanobar made of aluminum and silicon are listed for fixed-fixed and camped-free boundary conditions. Table 4 shows that 1) since the frequency ratios represent values less than one for both silicon and aluminium nanobars with fixed-fixed and fixed-free boundary conditions, it can be concluded that the surface energy and the nonlocal parameter have a decreasing effect on the natural longitudinal frequencies of the nanobar; 2) The classical frequencies of the nanobar and the nonlocal parameter effect depend on the frequency number and the type of boundary condition, but the effect of the surface energy components on the natural longitudinal frequencies of the nanobar is independent of the boundary condition type. In other words, the surface energy reduces all the nanobar classical frequencies with the same ratio for all boundary condition types; 3) the frequency ratios of aluminum and silicon nanobars are different when the surface energy effect is considered, but this is not the case for the frequency ratios when the nonlocal parameter effect is considered. This indicates that the effect of surface energy components on the natural longitudinal frequencies depends on the values of the mechanical properties of the nanobar while it is the other way round for the dependency of the nonlocal parameter effect on the values of the mechanical properties of the nanobar.

To consider the effects of another factor on natural longitudinal frequencies of nanobars, in Table 5, the fundamental frequencies and frequency ratios are listed for various values of the nanobar length. What Table 5 shows are exactly the results previously observed in Table 4. In summary, it can be stated that the effect of the surface energy components on the natural frequencies for various values of the nanobar length is independent of the boundary condition type while the nonlocal parameter effect depends on not only the nanobar length but also on the boundary condition type. The surface energy and the nonlocal parameter reduce the classical frequencies of the nanobar by the same ratio and different ratio, respectively. The other point of Table 5 is that the nonlocal parameter effect is independent of the nanobar material, while this is not the case for the surface energy effect. It is worth noting that the longitudinal frequencies of the nanobar are dependent on its length, and the larger the length of the nanobar, the lower the frequency. This can also be concluded from Eqs. (28) and (29) previously presented for natural frequencies of nanobar. This is because the parameter of the nanobar length is the denominator in the mentioned relations.

In Table 6, the other geometric factor of the nanobar, radius, is considered. To this end, the fundamental frequencies and frequency ratios of nanobar made of aluminum and silicon and for the fixed-fixed and fixed-free boundary conditions are given for various values of the nanobar radius. Table 6 reports different results than those reported in Tables 4 and 5. The results presented in Table 6 are: 1) for various values of the nanobar radius, the surface energy reduces the natural frequencies of nanobar by the same ratio for both boundary condition types, and its effect depends on the nanobar material while this is the opposite if the nonlocal parameter effect; 2) although the values of the classical frequencies are independent of the nanobar radius, the frequencies incorporating the surface energy effect depends on the value of the nanobar radius. The larger the nanobar radius, the lower the surface energy effect on the longitudinal frequencies. This decrease in the surface energy effect is due to the fact that by increasing the radius, the energy stored at the surface increases by a lower rate in comparison with the energy stored in the volume.

In the following, the point was examined that between two parameters (surface Lamé constants and surface density) affecting the natural longitudinal frequencies of the nanobar how the influence of each parameter is. For this purpose, in Fig. 2, variations of the fundamental frequency ratio versus the nanobar radius are plotted for various cases. The first result concluded from Fig. 2 is that the effects of the surface energy parameters on the longitudinal frequencies are not the same. Even the type of effect (increasing or decreasing) of the surface energy parameters is not the same. For silicon nanobars, both surface Lamé constants and surface density have a decreasing effect, and when their effects are simultaneously considered, they have a higher reducing effect than the one considered separately. Another interesting point is that however, the surface density has a much smaller value than the surface Lamé constants (refer to Table 3), but its decreasing effect is more. Nevertheless, for aluminum nanobars, the surface Lamé constants have an increasing effect, and the surface density has a decreasing effect.

Table 4. The first four classic frequencies and frequency ratios of aluminum and silicon nanobars with fixed-fixed and fixed-free boundary conditions (L=10 nm).

$\mu\ (nm^2)$	Frequency number	Aluminum		Silicon	
		Classic frequency (THz)	FR	Classic frequency (THz)	FR
			Fixed-Fixed		
0.0	1	1.5996	0.9042	2.9572	0.8420
	2	3.1992	0.9042	5.9145	0.8420
	3	4.7989	0.9042	8.8717	0.8420
	4	6.3985	0.9042	11.8289	0.8420
2.0 ($\tau_0 = \lambda_0 = \mu_0 = \rho_0 = 0$)	1	1.5996	0.9139	2.9572	0.9139
	2	3.1992	0.7475	5.9145	0.7475
	3	4.7989	0.6001	8.8717	0.6001
	4	6.3985	0.4904	11.8289	0.4904
2.0	1	1.5996	0.8263	2.9572	0.7694
	2	3.1992	0.6759	5.9145	0.6294
	3	4.7989	0.5426	8.8717	0.5053
	4	6.3985	0.4434	11.8289	0.4129
			Fixed-Free		
0.0	1	0.7998	0.9042	1.4786	0.8420
	2	2.3994	0.9042	4.4359	0.8420
	3	3.9991	0.9042	7.3931	0.8420
	4	5.5987	0.9042	10.3503	0.8420
2.0 ($\tau_0 = \lambda_0 = \mu_0 = \rho_0 = 0$)	1	0.7998	0.9762	1.4786	0.9762
	2	2.3994	0.8321	4.4359	0.8321
	3	3.9991	0.6691	7.3931	0.6691
	4	5.5987	0.5409	10.3503	0.5410
2.0	1	0.7998	0.8827	1.4786	0.8219
	2	2.3994	0.7524	4.4359	0.7006
	3	3.9991	0.6050	7.3931	0.5634
	4	5.5987	0.4891	10.3503	0.4554

Table 5. The fundamental frequency and frequency ratio of aluminum and silicon nanobars with fixed-fixed and fixed-free boundary conditions for various values of nanobar length.

$\mu\ (nm^2)$	Length (nm)	Aluminum		Silicon	
		Classic frequency (THz)	FR	Classic frequency (THz)	FR
			Fixed-Fixed		
0.0	10	1.5996	0.9042	2.9572	0.8420
	15	1.0664	0.9042	1.9715	0.8420
	20	0.7998	0.9042	1.4786	0.8420
	25	0.6398	0.9042	1.1829	0.8420
2.0 ($\tau_0 = \lambda_0 = \mu_0 = \rho_0 = 0$)	10	1.5996	0.9139	2.9572	0.9139
	15	1.0664	0.9588	1.9715	0.9588
	20	0.7998	0.9762	1.4786	0.9762
	25	0.6398	0.9846	1.1829	0.9846
2.0	10	1.5996	0.8263	2.9572	0.7694
	15	1.0664	0.8670	1.9715	0.8073
	20	0.7998	0.8827	1.4786	0.8219
	25	0.6398	0.8902	1.1829	0.8290
			Fixed-Free		
0.0	10	0.7998	0.9042	1.4786	0.8420
	15	0.5332	0.9042	0.9857	0.8420
	20	0.3999	0.9042	0.7393	0.8420
	25	0.3199	0.9042	0.5914	0.8420
2.0 ($\tau_0 = \lambda_0 = \mu_0 = \rho_0 = 0$)	10	0.7998	0.9762	1.4786	0.9762
	15	0.5332	0.9892	0.9857	0.9892
	20	0.3999	0.9939	0.7393	0.9939
	25	0.3199	0.9961	0.5914	0.9961
2.0	10	0.7998	0.8827	1.4786	0.8219
	15	0.5332	0.8944	0.9857	0.8329
	20	0.3999	0.8987	0.7393	0.8368
	25	0.3199	0.9006	0.5914	0.8387

Table 6. The fundamental frequency and frequency ratios of aluminum and silicon nanobars with fixed-fixed and fixed-free boundary conditions for various values of the nanobar radius.

$\mu\,(nm^2)$	Radius (nm)	Aluminum Classic frequency (THz)	Aluminum FR	Silicon Classic frequency (THz)	Silicon FR
			Fixed-Fixed		
0.0	1	0.1600	0.9042	0.2957	0.8420
	5	0.1600	0.9760	0.2957	0.9643
	10	0.1600	0.9876	0.2957	0.9819
2.0 ($\tau_0 = \lambda_0 = \mu_0 = \rho_0 = 0$)	1	0.1600	0.9990	0.2957	0.9990
	5	0.1600	0.9990	0.2957	0.9990
	10	0.1600	0.9990	0.2957	0.9990
2.0	1	0.1600	0.9033	0.2957	0.8411
	5	0.1600	0.9750	0.2957	0.9634
	10	0.1600	0.9866	0.2957	0.9809
			Fixed-Free		
0.0	1	0.0800	0.9042	0.1479	0.8420
	5	0.0800	0.9760	0.1479	0.9643
	10	0.0800	0.9876	0.1479	0.9819
2.0 ($\tau_0 = \lambda_0 = \mu_0 = \rho_0 = 0$)	1	0.0800	0.9998	0.1479	0.9998
	5	0.0800	0.9998	0.1479	0.9998
	10	0.0800	0.9998	0.1479	0.9998
2.0	1	0.0800	0.9040	0.1479	0.8418
	5	0.0800	0.9758	0.1479	0.9641
	10	0.0800	0.9874	0.1479	0.9816

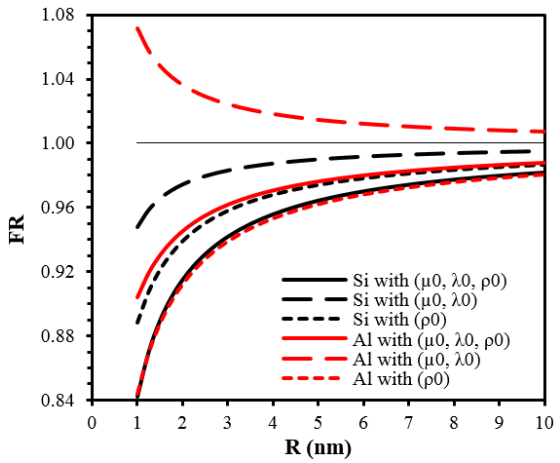

Fig. 2. Variations of fundamental FR versus the nanobar radius for various cases of surface energy effect.

When the effects of two factors are simultaneously considered a lower decreasing effect is achieved, which is less than the decreasing effect of the surface density parameter. This denotes that the decreasing effect of the surface density is dominant over the increasing effect of the surface Lamé constants. Due to the difference in the sign of the surface Lamé constants, it could be concluded that if the sign is positive, it has an increasing effect on the natural longitudinal frequencies, while it is the other way round for the negative sign of the surface Lamé constants.

In the final section of this study, the steady-state response of the middle of the fixed-fixed nanobar made of aluminum and silicon is shown in Fig. 3. In Fig. 3, the length and radius of the nanobar are considered to be 10 and 1 nm, respectively, and the following non-dimensional parameters are used:

$$U = u * \left(\frac{(EA)_{eq}}{4L^2\tau_0}\right); \quad T = t * \left(\frac{\pi c}{L}\right) \tag{38}$$

Fig. 3 shows that the surface energy changes the steady-state response of the nanobar longitudinal dynamic behavior, and its effect depends on the material of the nanobar, the values of the surface energy parameters and their signs.

4. Conclusion

In this paper, the dynamic behavior of the nanobar is considered analytically by considering the effects of surface energy components and nonlocal parameter for fixed-fixed and fixed-free boundary conditions. Due to considering the surface energy effect, the nanobar governing equation of motion is obtained inhomogeneous. For this reason, the homogeneous response is obtained first, and then the inhomogeneous response is calculated. The following results could be highlighted in the present study:

- The surface energy has a decreasing effect on the natural longitudinal frequencies of the nanobar, and its effect is independent of the frequency number, the length of the nanobar, and the type of boundary condition. However, the nanobar frequencies depend on the frequency number, the length of the nanobar, and the type of boundary condition.

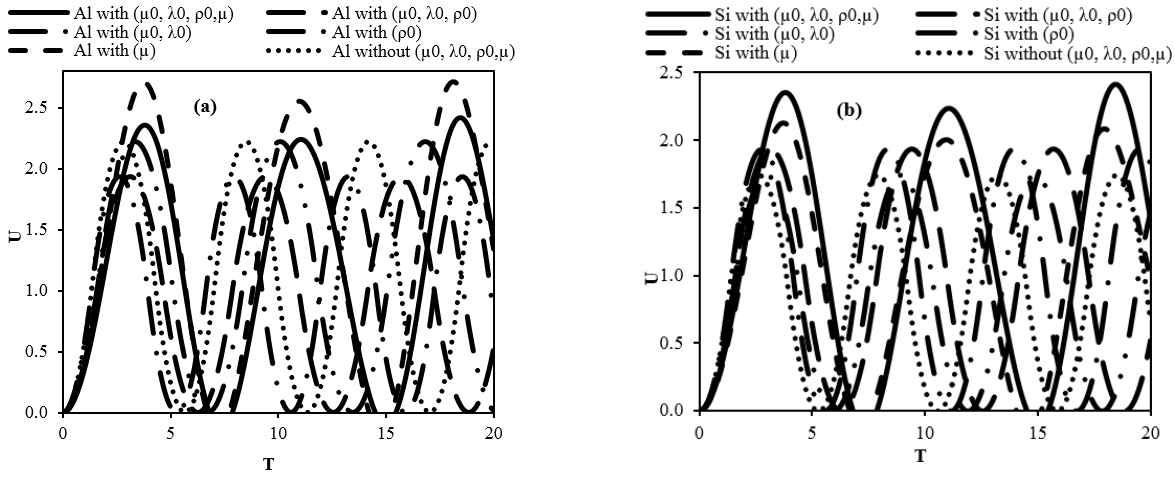

Fig. 3. The steady-state response of the middle of nanobeam with the fixed-fixed boundary condition, a) aluminum, b) silicon.

- The nonlocal parameter has a decreasing effect on the natural longitudinal frequencies of the nanobars, and its effect depends on the type of boundary condition, frequency number, and the length of nanobar.
- The surface energy and the nonlocal parameter do not affect the longitudinal mode shapes of the nanobar.
- Between the surface energy parameters, the surface stress does not affect the natural longitudinal frequencies, but it is effective on the overall dynamic response of the nanobar.
- Values of the natural longitudinal frequencies and the nonlocal parameter effect are independent of the nanobar radius, but the decreasing effect of the surface energy depends on the nanobar radius.
- When effects of the surface energy parameters are separately considered, it is observed that the surface density always has a decreasing effect, and its influence is greater than the that of the surface Lamé constants. In other words, the effect of the surface density is dominant over the effect of the surface Lamé constants. However, the effect of the surface Lamé constants depends on their sign. The surface Lamé constants with a positive sign have an increasing effect, but it is the other way round for the surface Lamé constants with the negative sign.

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