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> Date: Sunday, 17 May, 2015, 03:50 AM
> ---
> 17-May-2015
>
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> MS#: TPS8337
> MS TITLE: Magnetic Reconstruction for ITER using the Analytical Solution of GSE and TEQ Code
> MS AUTHORS: A. Naghidokht, R. Khodabakhsh, M. Ghoranneviss, A. Salar Elahi*
> MS TYPE: 6 Plasma Diagnostics
>
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Magnetic Reconstruction for ITER using the analytical solution of GSE and TEQ code

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Abstract: All plasma processes including linear and early nonlinear stages of MHD instabilities, transport and plasma flows, waves, micro-instabilities and turbulence, represent different kinds of deviations from MHD equilibrium and thus require accurate calculations of equilibrium configurations. The simplest useful mathematical model to describe equilibrium in fusion plasmas is achieved by combining magneto hydrodynamic (MHD) equations with Maxwell's equations. The final result is the Grad-Shafranov (GS) equation. Analytical solutions to the GS equation are an aid to theoretical investigations into plasma equilibrium, stability and transport in axisymmetric plasmas. In this paper we represent special analytical solution for GS equation and also simulation of equilibrium by TEQ code for ITER. Comparing between these two methods shows that simulation by TEQ has better results specifically for clearness of the x-point.

Keywords Equilibrium. Grad-Shafranov equation. Poloidal magnetic flux. ITER. TEQ code.

Introduction

Plasma equilibria lay at the fundamental level of magnetic confinement studies. All plasma processes including linear and early nonlinear stages of MHD instabilities, transport and plasma flows, waves, micro-instabilities and turbulence, represent different kinds of deviations from MHD equilibrium and thus require accurate calculations of equilibrium configurations [1]. Codes for numerical solution of many problems of tokamak physics (following particle orbits, investigating MHD and low frequency instabilities, simulating wave propagation for heating and current drive and so on) require the knowledge of the equilibrium plasma configuration in a continuously differentiable form. In the case of axisymmetry, the equations which define the MHD equilibrium,

$$\boldsymbol{\nabla}\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}\qquad,\boldsymbol{\nabla}.\mathbf{B}=0\qquad,\frac{1}{c}\mathbf{J}\times\mathbf{B}=\boldsymbol{\nabla}p$$

Are conveniently summarized by a single partial differential equation for the poloidal magnetic flux as a function of position in the poloidal plane, known as the Grad-Shafranov (GS) equation [2,3]. The GS quation for axisymmetric stationary ideal magneto hydrodynamics (MHD) flows has been considered very intensively within the last decades. Although analytical solutions of this equation are known [4-7], they are restricted to rather special pressure and current profiles. Exact solutions to the GSE are an aid to theoretical investigations into plasma equilibrium, stability and transport in axisymmetric plasmas. The GS equation is, in general, a nonlinear partial differential equation (PDE) and its solution rely on numerical methods. However for some choices of the arbitrary functions, the equation becomes linear and separable and the boundary value problem can be solved by superposition of independent solutions. Because of the nonlinearity of the GS equation, all numerical methods for equilibrium calculations are iterative by nature. Practically all existing codes use a simple iteration method for solving the nonlinear GS equation. Although this approach is sufficient in most of the situations, there is still a demand for fast methods which can be used for extensive calculations of magnetic configurations in transport codes for plasma stability studies, equilibrium reconstructions from experimental measurements and for simulations of equilibrium control in real machines.

In this paper, in section 1 we represent special analytical solution for GS equation and then we apply the results to ITER. In section 2 we introduce equilibrium code, TEQ, and based on this code we calculate equilibrium configuration for ITER. Following this section, a brief comparison between these two approaches is presented. In section 3 we briefly discuss about results.

Special Analytical Solution for GS equation and application to ITER

The magnetic field is related to the poloidal flux Ψ by:

$$B_\varphi=\frac{F(\Psi)}{R}\quad,\quad\mathbf{B}_p=\frac{\boldsymbol{\nabla}\Psi\times\mathbf{e}_\varphi}{R}\tag{1}$$

where B_φ and $\mathbf{B}_p$ are toroidal and poloidal magnetic fields. We choose the free functions $p(\Psi)$ and $F(\Psi)$ to be quadratic in Ψ [8]:

$$p=p_{axis}\left(\frac{\Psi^2}{\Psi_{axis}^2}\right)\quad,\quad F^2=R_0^2B_0^2\left[1+b_{axis}\left(\frac{\Psi^2}{\Psi_{axis}^2}\right)\right]\tag{2}$$

Here Ψ_{axis} , p_{axis} and b_{axis} are constants related to the values of Ψ, p and F on axis and R_0 and B_0 are the major radius and vacuum toroidal field at the geometric center of the plasma. With these choices and by separation of variables and also for an up/down symmetric configuration (like ITER), finally we have:

$$\psi=\sum_m X_m(\rho)Y_m(y)\ ;\quad Y_m(y)=\cos(k_m y)\ \ ,\ \ X_m(\rho)=Im[a_m W_{\lambda_m,\mu}(\rho)+b_m M_{\lambda_m,\mu}(\rho)]\tag{3}$$

Where k_m is the m[th] separation constant, which can be real or imaginary and we should determine it, $\lambda_m = -i\frac{\gamma - k_m^2}{4\epsilon\sqrt{\alpha}}$, $\Psi = \Psi_{axis}\psi$, $R^2 = R_0^2 x$, $z = ay$, $\epsilon = \frac{a}{R_0}$, $\gamma = (\frac{aR_0B_0}{\Psi_{axis}})^2 b_{axis}$, $\alpha = (\frac{aR_0B_0}{\Psi_{axis}})^2 \beta_{axis}$, $\beta_{axis} = \frac{2\mu_0 p_{axis}}{B_0^2}$, $x = \frac{-i\epsilon\rho}{\sqrt{\alpha}}$ and the solutions of $X_m(\rho)$ are Whittaker functions [9]. The a_m and b_m are unknown expansion coefficients that must be determined. Both ρ and λ_m are purely imaginary quantities while X_m must be purely real, then we only keep imaginary parts of Whittaker functions. For our model the Whittaker parameter is $\mu = \frac{1}{2}$. With maintaining three terms in the summation of ψ, we have:

$$\psi(\rho, y) = \sum_{m=1}^{3}[\, Im[a_m W_{\lambda_m,\mu}(\rho) + b_m M_{\lambda_m,\mu}(\rho)]]\, cos(k_m y) \tag{4}$$

There are six unknown expansion coefficients (i.e. the a_m,b_m) and three unknown separation constants (i.e. the k_m) that must be determined. These coefficients are determined with specific conditions. These conditions are:

$$\Psi(R_0 + a, 0) = 0 \quad , \Psi(R_0 - a, 0) = 0 \quad , \Psi(R_0 - \delta a, \kappa a) = 0 \quad , \Psi_R(R_0 - \delta a, \kappa a) = 0 \quad , \Psi(R_{\text{axis}}, 0) = \psi_{\text{axis}} , \Psi_R(R_{\text{axis}}, 0) = 0 \, , \; \frac{1}{R_c} \equiv -\frac{(1-\hat{\delta})^2}{\kappa^2 a} \tag{5}$$

where $\hat{\delta} = \sin^{-1}\delta$, δ is triangularity, a is minor radius and κ is elongation. Also R_c is curvature radius for inboard midplane. Based on empirical experience, we define separation constants as:

$$k_1 = 0 \;\; , \; k_2 = i\hat{k}_2 \;\; k_3 = \frac{\pi}{\kappa}\hat{k}_3 \tag{6}$$

with $\hat{k}_2$, $\hat{k}_3$ real and of order unity. We will assume that appropriate values for $\hat{k}_2$, $\hat{k}_3$ are given once the plasma geometry has been chosen [8].

The primary inputs for the solution procedure are the inverse aspect ratio ϵ, the elongation κ, the triangularity δ, and the parameters α and γ. The secondary input parameters are the major radius R_0, the vacuum toroidal field B_0 and the toroidal current I. For ITER, these input data are given in Table1 [8]. The final form of the solution for ψ is given by:

$$\psi(\rho, y) = \left(a_1 W_{\lambda_1,\frac{1}{2}}(\rho) + b_1 M_{\lambda_1,\frac{1}{2}}(\rho)\right) + \left(a_2 W_{\lambda_2,\frac{1}{2}}(\rho) + b_2 M_{\lambda_2,\frac{1}{2}}(\rho)\right) cos(k_2 y) + \left(a_3 W_{\lambda_3,\frac{1}{2}}(\rho) + b_3 M_{\lambda_3,\frac{1}{2}}(\rho)\right) cos(k_3 y) \tag{7}$$

The parameters for calculation of equilibrium for ITER are: $= -0.5$, $k_2 = 0.90i$, $k_3 = 1.82$ and $\alpha = 4.48$ [8]. The resulting flux surfaces for ITER are illustrated in Fig. 1.

Table 1: Characteristic data for ITER [8].

Parameters	Value	Parameters	Value
$\boldsymbol{\epsilon}$, inverse aspect ratio	0.32	R_0, Major radius (m)	6.2
$\boldsymbol{\kappa}$, elongation	1.8	a, Minor radius (m)	2
$\boldsymbol{\delta}$, triangularity	0.45	B_0, toroidal field (T)	5.3
$\boldsymbol{q_{axis}}$, Safety factor on axis	1	I, toroidal current (MA)	10.1

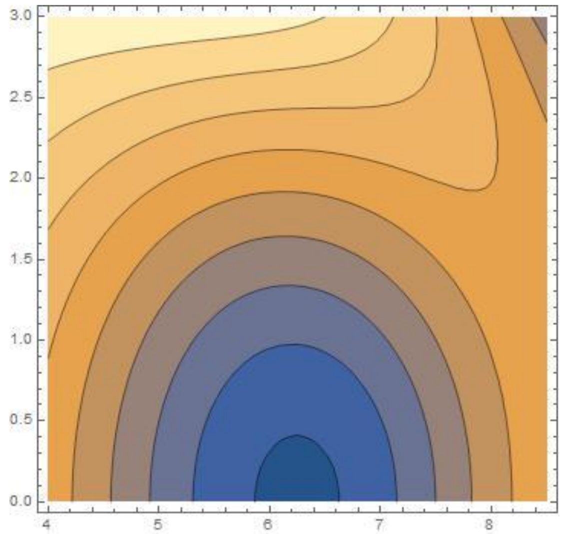

Fig. 1 Flux surfaces for ITER (due to up-down symmetry, only half of the plasma is shown).

Also we calculate toroidal current density (J_φ) and pressure (normalized pressure) profiles versus the major radius along the midplane ($z = 0$) for ITER in Fig. 2. The vanishing of ∇p and J_φ at the edge, combined with the linear dependence on Ψ , produces peaked pressure and current profiles. Although some unknown parameters in this method are tuned with experiment but all profiles are quite realistic.

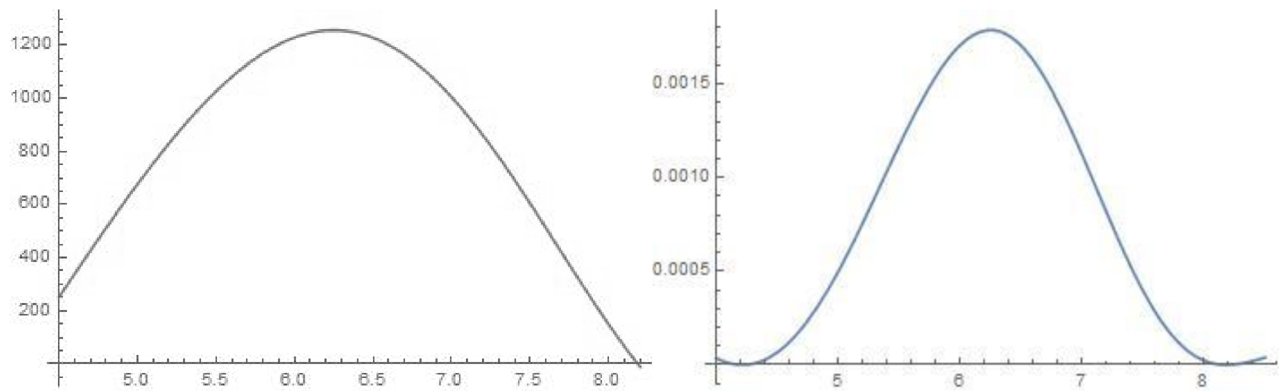

Fig. 2 Toroidal current density (in kA/m^2), left, and normalized pressure, right, profiles versus the major radius along the midplane ($z = 0$) for ITER.

Simulation of equilibrium configuration for ITER by TEQ code

In this section we introduce TEQ code for simulation of equilibrium configuration. One of the methods which have been proposed to solve GS equation, is Green's function method as an analytical solution to GS equation. For this, we have:

$$\Delta^* \psi = -\mu_0 r J_t \tag{8}$$

In which $\psi = rA_\varphi$ is the magnetic poloidal flux, and A_φ is the toroidal component of the magnetic vector potential. Also, J_t is the toroidal current density and Δ^* is the elliptic Grad-Shafranov operator which is defined as:

$$\Delta^{*}= R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right)+\frac{\partial^2}{\partial Z^2} \tag{9}$$

In equation 8, we disregard the inherent dependence of J_t on the poloidal flux ψ, making the GS equation a linear differential equation. From a system engineering point of view, GS equation represents a Linear Time Invariant (LTI) system whose impulse response is given by its associated Green's function. Here in order to find flux function, we seek solutions of the form:

$$\psi(r,z)=\int_{-\infty}^{+\infty}\int_{0}^{+\infty} G(\mathbf{r},\mathbf{r}')J_t(\mathbf{r}')dr'dz' \tag{10}$$

Where $\mathbf{r}=(r,z)$ is the two-dimensional position vector on the constant poloidal plane and $G(\mathbf{r},\mathbf{r'})$ is referred to as the Green's function, obtained through the solution of the following equation:

$$\Delta^{*}\psi=\mu_0\delta(\mathbf{r}-\mathbf{r}')=\mu_0\delta(r-r')\delta(z-z') \tag{11}$$

Where $\delta(.)$ being the Dirac's delta function. The poloidal flux function ψ can be accurately obtained by through the Green's function formalism once the toroidal current density profile is known. As GS equation numerically is a non-linear partial differential equation (PDE), the use of numerical solution is inevitable for description of axisymmetric plasma equilibria. Various numerical methods have been proposed to solve GS equation, which could be found in the literature. The Finite Element Method (FEM) is the most popular general purpose technique for computing accurate solutions to PDEs, which we hereby exploit to solve GSE. The family of FEMs may be divided into Galerkin and variational approaches, in both of which the solution is expanded on a set of eigenfunctions. Equation 8 may be regarded as an Euler-Ostogradskii equation of the functional:

$$\Pi(\psi)=\iint\left(\frac{1}{2r}|\nabla\psi|^2-\mu_0\psi J_t\right)drdz \tag{12}$$

where the integration is taken over a domain Ω in the two-dimensional (r,z) plane and $\nabla=\frac{\partial}{\partial r}\hat{r}+\frac{\partial}{\partial z}\hat{z}$ is the two-dimensional gradient. The basic idea of the FEM is to make a piecewise approximation, that is the solution of a problem is achieved by dividing the region of interest into small regions called elements, and approximating the solution over each element by simple function with prescribed forms. The functions used to represent the behavior of the solution within an element are called interpolation functions; the simplest choice is linear dependence to coordinates referring to first-order elements. For example, the simplex element in two dimensions is a triangle with three nodes (corners). Nodes are usually shared by more than one element and it is desirable to find the nodal values of unknown functions through a set of algebraic operations which simultaneously extermize equation 12. In Fig. 3 we calculate equilibrium for ITER by TEQ code. TEQ is Livermore's toroidal equilibrium code that was extracted from CORSICA code [10] as an NTCC module and can be used both for prescribed boundary and free boundary equilibria. Free boundary TEQ equilibrium solver applied to generate new equilibrium solution as plasma profiles evolve, including scrape-off region. It has been extensively used to study the nature of operating scenarios and system limitations in ITER and then we use this code for simulation of equilibrium in ITER. It is found that the TEQ code produces the smallest residual error. In TEQ code various coils of special tokamak are simulated by finite number of filaments carrying currents and poloidal flux function is calculated with considering of different regions in the system [11,12]. Comparing to Fig. 1, we can see that simulation by TEQ has better result than the method of previous section specifically for the x-point that can be seen clearly in Fig. 3.

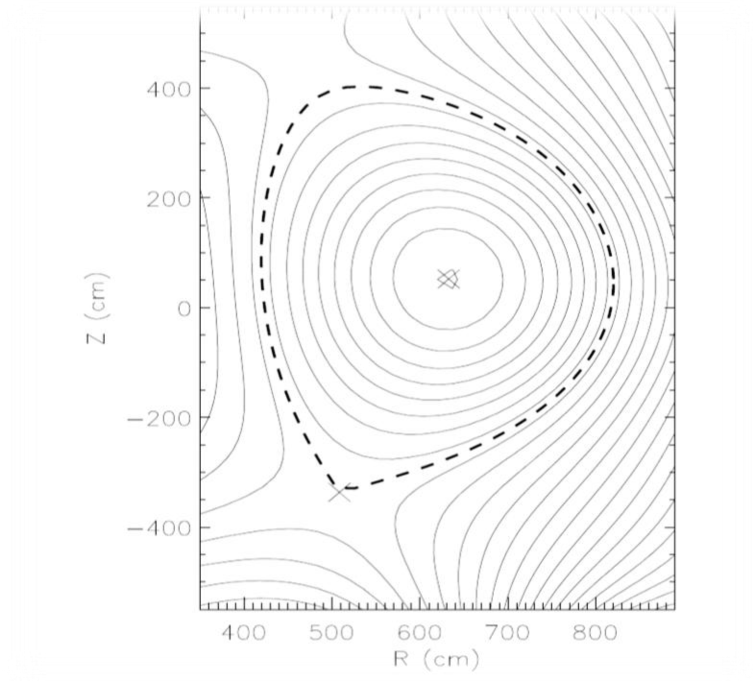

Fig. 3 ITER equilibrium calculated by TEQ code. The dashed line is the separatix and the crosses are the O-point and the X-point.

Discussion

The solutions of Grad-Shafranov (GS) equation analytically can be used for theoretical studies of plasma equilibrium, transport and magneto hydrodynamic stability. In this paper, we choose specific choices for free functions (p and F) to be quadratic in ψ and derive a special analytic solution for GS equation. This solution is applied to ITER and poloidal magnetic flux, toroidal current density and pressure profiles are calculated. Although some unknown parameters in this method are tuned with experiment but results of this method are quite realistic. Also by TEQ code, equilibrium is calculated for ITER. Comparison between these two methods shows better results for simulation by TEQ specifically for clearness of the x-point.

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