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A New Perspective on Protection of Nuclear Reactor Surfaces from High Energy Plasma Irradiation by Equilibrium Reconstruction

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Abstract- Ignition is the point at which a nuclear fusion of hydrogen isotopes reaction becomes self-sustaining. This occurs when the energy being given off by the fusion reactions heats the fuel mass more rapidly than various loss mechanisms cool it. At this point, the external energy needed to heat the fuel to fusion temperatures is no longer needed. As the rate of fusion varies with temperature, the point of ignition for any given machine is typically expressed as a temperature. On the other hand, energy confinement time is expressed by an equilibrium state. Analytical solutions of Grad-Shafranov equation (GSE) can be used for theoretical studies of plasma equilibrium, transport and Magnetohydrodynamic stability of tokamaks. In this paper we have presented two families of exact solutions. With applying these solutions to IR-T1 tokamak, a small, air core, low beta and large aspect ratio tokamak with a circular cross section, we calculated poloidal magnetic flux. Due to the generality and high accuracy of the second exact solution for all of the magnetic configurations of interest, the result of this solution for IR-T1 tokamak is good (tokamak-relevant) equilibrium compare to the first exact solution for this tokamak. We intend to use these analytical solutions as benchmark of numerical equilibrium codes.

Index Terms: The Grad-Shafranov equation, Poloidal magnetic flux, IR-T1 tokamak

I. INTRODUCTION

For axially symmetric configurations, where φ is the ignorable angle in the cylindrical coordinate system (r,φ,z), Maxwell's equations together with the force balance equation reduce, for stationary and ideally conducting plasmas, to the scalar partial differential equation named the Grad–Shafranov equation (GSE) [1-10]:

$$\left(\frac{\partial^2\psi}{\partial r^2} \quad \frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}\right)= \mu_0 R^2 p'(\psi) + FF'(\psi) = \mu_0 R j_\varphi \ , \tag{1}$$

where j_φ is the toroidal current density, the stream function ψ is the poloidal magnetic flux per radian, $F = RB_\varphi = \frac{\mu_0 I_{pol}(\psi)}{2\pi}$ where I_{pol} is the poloidal current, $p'(\psi) = \frac{dp}{d\psi}$ and $FF'(\psi) = \frac{d(\frac{1}{2}F^2)}{d\psi}$ that $p(\psi)$ and $F(\psi)$ are in the set of ideal Magnetohydrodynamic (MHD) arbitrary functions of the ψ function. Analytical solutions of the GSE are very useful for theoretical studies of plasma equilibrium, transport, and magnetohydrodynamic stability. The literature on exact solutions to the Grad–Shafranov equation is extensive. These solutions can be used also as a benchmark of numerical codes, but existing exact solutions are very restricted in a variety of allowed current density profiles. The simplest analytical solution to the inhomogeneous GSE is the well-known Solovev equilibrium [11-20] and corresponds to source functions linear in ψ. Equilibria of this type have been extensively used for equilibrium, transport, and stability studies. For the same Solov'ev equilibrium case, by expanding the solution of the homogeneous equations in a polynomial form in r (of fourth degree) and z (of second degree), and assuming an up–down symmetry, it is possible to describe the plasma shape by four parameters. By using source functions quadratic in ψ for the GSE, a class of exact analytical solutions are found [21-24]. An exact solution of the large-aspect-ratio approximation with an additional assumption of a simple relation between the magnetic flux and the current density was constructed. A new family of solutions is presented, where the plasma pressure is linear in ψ, while the squared poloidal current has both, a quadratic and a linear ψ term. Yet interest in new techniques for solving the equation remains strong. The paper is organized as follows: In section 1, a family of exact analytical solution of the GSE is presented by considering that the current density profile has four free parameters, the most complicated dependence on the flux function ψ which still maintains the linear character of the equation with partial derivatives. Also, we present an extended analytic solution to the GS equation with Solovev profiles which possesses sufficient freedom to describe a variety of magnetic configurations. Section 2 is devoted to application of these two analytical solutions to IR-T1 tokamak, which is a small, air core, low β and large aspect ratio tokamak with a circular cross section, and comparison of the results. Comments and discussions are contained in section 3.

II. TWO FAMILIES OF ANALYTICAL SOLUTIONS

It seems that a general property of the analytic solutions is that they contain only a very few terms, thereby making them attractive from a theoretical analysis point of view. In this section we present two families of analytical solutions of the GSE.

A. First exact solution

A family of exact analytical solution of the GSE is presented by considering that the current density profile has four free parameters, the most complicated dependence on the flux function Ψ ($= 2\pi\psi$)which still maintains the linear character of the equation with partial derivatives. Considering the pressure and the poloidal current profiles dependencies on Ψ of the form [10]:

$$p(\Psi) = \bar{a}\Psi^2 + \bar{b}\Psi \quad (2)$$

$$F^2(\Psi) = \bar{\alpha}\Psi^2 + \bar{\beta}\Psi + F_0^2 \quad (3)$$

The GSE and the plasma current density looks like:

$$\frac{\partial^2\Psi}{\partial r^2} \quad \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{\partial^2\Psi}{\partial z^2} = \quad (ar^2 + \alpha)\Psi \quad (br^2 + \beta), \quad (4)$$

$$j_\varphi(r,z) = \frac{1}{2\pi\mu_0}\left[\left(ar + \frac{\alpha}{r}\right)\Psi + br + \frac{\beta}{r}\right], \quad (5)$$

where $a = 8\pi^2\mu_0\bar{a}$, $b = 4\pi^2\mu_0\bar{b}$, $\alpha = \mu_0^2\bar{\alpha}$ and $\beta = \frac{\mu_0^2}{2}\bar{\beta}$ are the four free parameters allowing to independently specify the plasma current I_{pl} , the poloidal beta β_{pol} , the internal inductance l_i and the safety factor at the magnetic axis q_{ax} or at the plasma boundary q_b . With converting the original inhomogeneous partial differential equation (PDE) into two problems, a homogeneous PDE and an inhomogeneous ordinary differential equation, we have the general solution as:

$$\Psi = \Psi_0 + \Psi_{nop} , \quad (6)$$

where $\Psi_o = R(r)Z(z)$ represents the general homogeneous solution, while Ψ_{nop} is any particular inhomogeneous solution. Depending on the sign of the a parameter, two possible families of analytical solutions have been deduced. After some computations that presented in Ref. [10] and by using the notation $\omega = kz$(k is an arbitrary constant) for both cases, $a > 0$ and $a < 0$ respectively, the general solution can be put in the following compact form:

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$$\Psi(r,z) = [C_1 F_1(\eta,r) + C_2 F_2(\eta,r)][C_3 \cos\omega + C_4 \sin\omega] + \Psi_{nop}(r) \quad , \tag{7}$$

where for $a < 0$:

$$F_1(\eta,r) = \sqrt{\ \ a}r^2 e^{-\sqrt{-a}r^2/2} \times 1F1(1+\eta;2;\ \sqrt{\ \ a}r^2)$$

$$F_2(\eta,r) =$$
$$\sqrt{\ \ a}r^2 e^{-\sqrt{-a}r^2/2}\left[\left(\frac{2}{\sqrt{-a}r^2} + \eta \log(\ \sqrt{\ \ a}r^2\right) \times 1F1\left(1+\eta;2;\ \sqrt{\ \ a}r^2\right) \quad \frac{1}{\sqrt{-a}r^2} \times 1F1\left(\eta;1;\ \sqrt{\ \ a}r^2\right) +$$
$$\sum_{n=0}^{\infty} \frac{(\eta)_{n+1}(\sqrt{-a}r^2)^n}{n!(n+1)!} \times \left(\sum_{r=1}^{n+1} \frac{1}{r-1+\eta} \quad 2\sum_{r=1}^{n} \frac{1}{r}\right)\right]$$

$$\Psi_{nop}(r) = \quad \frac{\beta}{\alpha} + \frac{\sqrt{-a}}{4} r^2 e^{-\frac{\sqrt{-a}r^2}{2}} \left(\frac{\beta}{\alpha} \quad \frac{b}{a}\right) \times \sum_{n=0}^{\infty} \frac{1}{n!(n+\tilde{a})} \times 2F1\left(2+n,1;n+1+\tilde{a};\ \frac{1}{2}\right)\left(\frac{\sqrt{-a}}{2} r^2\right)^n] \ . \tag{8}$$

And for > 0:

$$F_1(\eta,r) = F_0\left(\eta, \frac{\sqrt{a}}{2} r^2\right) \quad , \quad F_2(\eta,r) = G_0\left(\eta, \frac{\sqrt{a}}{2}\right)$$

$$\Psi_{nop}(r) = \quad [\ \frac{\beta}{\alpha} + i\frac{\sqrt{a}}{4} r^2 e^{-\frac{i\sqrt{a}r^2}{2}} \left(\frac{\beta}{\alpha} \quad \frac{b}{a}\right) \times \sum_{n=0}^{\infty} \frac{1}{n\left(n+1+i\frac{\alpha}{4\sqrt{a}}\right)} \times 2F1\left(2+n,1;n+2+i \quad \frac{\alpha}{4\sqrt{a}};\frac{1}{2}\right)\left(i\frac{\sqrt{a}}{2} r^2\right)^n] \ , \tag{9}$$

where $\eta \equiv \frac{k^2-\alpha}{4\sqrt{-a}}$ $(\eta \neq 0,1)$, $1F1(1+\eta;2;\ \sqrt{\ \ a}r^2)$ and $2F1\left(2+n,1;n+1+\tilde{a};\ \frac{1}{2}\right)$ are two types of hypergeometric functions and $(\eta)_k$ designates the Pochhammer symbol as: $(\eta)_0 = 1,\ (\eta)_k = \eta(\eta+1)\ldots(\eta+k \quad 1), k = 1,2,\ldots$. Also we have $\tilde{a} = 1 \quad \frac{\alpha}{4\sqrt{-a}}$ and $F_0\left(\eta, \frac{\sqrt{a}}{2} r^2\right)$ and $G_0\left(\eta, \frac{\sqrt{a}}{2} r^2\right)$ are Coulomb wave functions with $L = 0$. In configurations with up/down symmetry, only terms in cosine retain. Then we can write Eq. (7) as follows:

$$\Psi(r,z) = [D_1 F_1(\eta,r)\cos\omega + D_2 F_2(\eta,r)\cos\omega] + \Psi_{nop}(r) \quad , \tag{10}$$

where $D_1 = C_1 C_3$ and $D_2 = C_2 C_3$. For a current point on the plasma boundary (r_i, z_i) we can write:

$$\Psi(r_i, z_i) = \sum_k [D_{1k} F_1(\eta_k, r_i)\cos\omega_{ki} + D_{2k} F_2(\eta_k, r_i)\cos\omega_{ki}] + \Psi_{nop}(r_i)\ . \tag{11}$$

Knowing the values of the a, b, α and β parameters, the constants D_{1k} and D_{2k} can be determined if both the value of the flux function ψ on the plasma boundary and the plasma contour are given. The parameters a, b, α and β are determined by using integral relations concerning the following global plasma parameters: the plasma current I_{pl}, the poloidal beta β_{pol}, the internal inductance l_i and the safety factor at the magnetic axis q_{ax} or at the plasma boundary q_b. By using Eq. (2) and (5) and from the definition relations, one obtains [10]:

$$a\iint \Psi(r,z) r\,dr\,dz + \alpha \iint \Psi(r,z)\frac{1}{r}dr\,dz + b\iint r\,dr\,dz + \beta \iint \frac{1}{r}dr\,dz = 2\pi\mu_0 I_{pl}$$

$$\frac{a}{2}\iint \Psi^2(r,z) r\,dr\,dz + b\iint \Psi(r,z) r\,dr\,dz = \frac{\iint r\,dr\,dz}{(\oint dl)^2} 2\pi^2\mu_0^2 I_{pl}^2 \beta_p \ ,$$

$$a\iint \Psi^2(r,z) r\,dr\,dz + \alpha \iint \Psi^2(r,z)\frac{1}{r}dr\,dz + b\iint \Psi(r,z) r\,dr\,dz + \beta \iint \Psi(r,z)\frac{1}{r}dr\,dz = \frac{\iint r\,dr\,dz}{(\oint dl)^2} 4\pi^2\mu_0^2 I_{pl}^2 l_i \ ,$$

$$aR_{ax}^2 \Psi_{ax} + \alpha\Psi_{ax} + bR_{ax}^2 + \beta = 2\sqrt{\alpha\Psi_{ax}^2 + 2\beta\Psi_{ax} + (F_0\mu_0)^2}/R_{ax}/q_{ax} \ . \tag{12}$$

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All the double integrals in the above written relations have to be performed over the total cross-section area of the plasma. By using these four relations we can determine the four parameters a, b, α and β. Together with the determined constants, D_{1k} and D_{2k}, our analysis is complete and now we can compute the function $\Psi(r,z)$.

B. Second exact solution

The GSE, Eq. (1) for $\Psi(r,z)$, can be put in a non-dimensional form through the normalization $R = R_0 x$, $z = R_0 y$ and $\Psi = \Psi_0 \psi$, where R_0 is the major radius of the plasma and Ψ_0 is an arbitrary constant. Then Eq. (1) becomes:

$$x\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial\psi}{\partial x}\right)+\frac{\partial^2\psi}{\partial y^2} = \quad \mu_0\frac{R_0^4}{\Psi_0^2}x^2\frac{dp}{d\psi} \quad \frac{R_0^2}{\Psi_0^2}F\frac{dF}{d\psi} \ . \quad (13)$$

The well-known choices for p and F corresponding to the Solov'ev profiles are given by:

$$\mu_0\frac{R_0^4}{\Psi_0^2}\frac{dp}{d\psi} = C\ , \qquad \frac{R_0^2}{\Psi_0^2}F\frac{dF}{d\psi} = A, \quad (14)$$

where A and C are constants. Since Ψ_0 is an arbitrary constant, one can, without loss in generality, choose it such that $A + C = 1$ (The special case $A + C = 0$ cannot occur for physical equilibria since it corresponds to a situation beyond the equilibrium limit where the separatrix moves onto the inner plasma surface). This is formally equivalent to the rescaling $\Psi_0^2 \rightarrow (A + C)\Psi_0^2$. Under these conditions, the GS equation with Solov'ev profiles can be written in the following dimensionless form:

$$x\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial\psi}{\partial x}\right)+\frac{\partial^2\psi}{\partial y^2} = (1 \quad A)x^2 + A\ . \quad (15)$$

The choice of A defines the β regime of interest for the configuration under consideration. The solution to Eq. (15) is of the form $\psi(x,y) = \psi_P(x,y) + \psi_H(x,y)$, where ψ_P is the particular solution and ψ_H is the homogeneous solution. The particular solution can be written as:

$$\psi_P(x,y) = \frac{x^4}{8} + A\left(\frac{1}{2}x^2\ln x \quad \frac{x^4}{8}\right). \quad (16)$$

The homogeneous solution satisfies:

$$x\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial\psi_H}{\partial x}\right)+\frac{\partial^2\psi_H}{\partial y^2} = 0\ . \quad (17)$$

A general arbitrary degree polynomial solution to this equation for plasmas with up-down symmetry has been derived in Ref. [4]. We retain only a finite number of terms in the possible infinite sum of polynomials. We truncate the series such that the highest degree polynomials appearing are R^6 and z^6 [11]. The full solution for up-down symmetric ψ including the most general polynomial solution for ψ_H satisfying Eq. (17) and consistent with our truncation criterion is given by:

$$\psi(x,y) = \frac{x^4}{8} + A\left(\frac{1}{2}x^2\ln x \quad \frac{x^4}{8}\right) + c_1\psi_1 + c_2\psi_2 + c_3\psi_3 + c_4\psi_4 + c_5\psi_5 + c_6\psi_6 + c_7\psi_7$$

$$\psi_1 = 1,\ \psi_2 = x^2,\ \psi_3 = y^2 \quad x^2\ln x,\ \psi_4 = x^4 \quad 4x^2y^2,\ \psi_5 = 2y^4 \quad 9x^2y^2 + 3x^4\ln x \quad 1\,2x^2y^2\ln x,\ \psi_6 = x^6 \quad 12x^4y^2 + 8x^2y^4,\ \psi_7 = 8y^6 \quad 140x^2y^4 + 75x^4y^2 \quad 15x^6\ln x + 180x^4y^2\ln x \quad 120x^2y^4\ln x\ , \quad (18)$$

Equation (18) is the desired exact solution to the GS equation that describes all the configurations of interest that possess up-down symmetry [11]. The unknown constants c_n are determined from as yet unspecified boundary constraints on ψ. For this purpose, consider first the case where the plasma surface is smooth. A good choice for these properties is to match the function and its first and second derivatives at three test points: the inner equatorial point, the outer equatorial point, and the high point (see Fig. 1 for the geometry). While this might appear to require nine free constants (i.e., three conditions at each of the three points), two are redundant because of the up-down symmetry. Although it is intuitively clear how to

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specify the function and its first derivative at each test point, the choice for the second derivative is less obvious. To specify the second derivatives we make use of a well-known analytic model for a smooth, elongated "D" shaped cross section, which accurately describes all the configurations of interest. The boundary of this cross section is given by the parametric equations:

$$x = 1 + \varepsilon \cos(\tau + \alpha \sin\tau), \quad y = \varepsilon\kappa \sin\tau \qquad (19)$$

where τ is a parameter covering the range $0 \le \tau \le 2\pi$. Also, $\varepsilon = \frac{a}{R_0}$ is the inverse aspect ratio, κ is the elongation, and $\sin\alpha = \delta$ is the triangularity. Using these parametric equations it is straightforward to evaluate the desired second derivatives at each of the three test points.

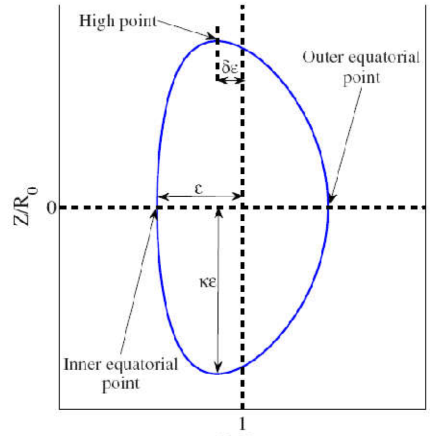

Fig.1, Definition of the geometric parameters

The seven geometric constraints are given below assuming that the free additive constant associated with the flux function is chosen so that $\psi = 0$ on the plasma surface (Then $\psi < 0$ in the plasma). Therefore we have:

$\psi(1+\varepsilon, 0) = 0$ Outer equatorial point, $\psi(1-\varepsilon, 0) = 0$ inner equatorial point,

$\psi(1-\delta\varepsilon, \kappa\varepsilon) = 0$ High point, $\psi_x(1-\delta\varepsilon, \kappa\varepsilon) = 0$ high point maximum,

$\psi_{yy}(1+\varepsilon, 0) = -N_1\psi_x(1+\varepsilon, 0)$ Outer equatorial point curvature,

$\psi_{yy}(1-\varepsilon, 0) = -N_2\psi_x(1-\varepsilon, 0)$ Inner equatorial point curvature,

$\psi_{xx}(1-\delta\varepsilon, \kappa\varepsilon) = -N_3\psi_y(1-\delta\varepsilon, \kappa\varepsilon)$ High point curvature (20)

The coefficients N_j are easily found from the model surface specified by Eq. (19) and can be written as:

$$N_1 = \left[\frac{d^2x}{dy^2}\right]_{\tau=0} = \frac{(1+\alpha)^2}{\varepsilon\kappa^2}; \quad N_2 = \left[\frac{d^2x}{dy^2}\right]_{\tau=\pi} = \frac{(1-\alpha)^2}{\varepsilon\kappa^2}; \quad N_3 = \left[\frac{d^2y}{dx^2}\right]_{\tau=\frac{\pi}{2}} = \frac{\kappa}{\varepsilon\cos^2\alpha}; \qquad (21)$$

For a given value of A, the conditions given by Eq. (20) reduce to a set of seven linear inhomogeneous algebraic equations for the unknown c_n. This is a trivial numerical problem. In final step and for determination of A, we can use the poloidal beta definition relation (based on our approach here) [11]:

$$\beta_p(\varepsilon,\kappa,\delta,A) = -2(1-A)\frac{C_p^2}{V}\left[\int \psi\, x\,dx\,dy\right]\left\{\int \frac{dx\,dy}{x}\left[A + (1-A)x^2\right]\right\}^{-2}, \qquad (22)$$

where C_p is the normalized poloidal circumference of the plasma surface and V is the normalized plasma volume, as given by:

$$C_p = \frac{1}{R_0}\oint dl_p = 2\int_{1-\varepsilon}^{1+\varepsilon}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx, \quad V = \frac{1}{2\pi R_0^3}\int d\mathbf{r} = \int x\,dx\,dy, \qquad (23)$$

with determination of A and then the unknown constants c_n, our analysis in now complete and we can apply it to the magnetic configurations of interest.

III. APPLICATION TO IR-T1 TOKAMAK

For Applying of these two analytical solutions to IR-T1 tokamak, the primary inputs are the inverse aspect ratio ε, the elongation κ, the triangularity δ, the poloidal beta β_p, the plasma current I_{pl}, the internal inductance l_i and the safety factor at the magnetic axis q_{ax} or at the plasma boundary q_b. For IR-T1 tokamak, these input data are depicted in Table 1 [9].

Table 1: Characteristic data for IR-T1

Parameters	Value	Parameters	Value
$\boldsymbol{\epsilon}$, inverse aspect ratio	0.2777	R_0, Major radius (m)	0.45
$\boldsymbol{\kappa}$, elongation	1	a, Minor radius (m)	0.125
$\boldsymbol{\delta}$, triangularity	0	β_t, toroidal beta	0.01
$\mathbf{q_{axis}}$, safety factor on axis	1	l_i, internal inductance	0.5
$\mathbf{B_0}$, toroidal field (T)	0.8	q , kink safety factor	2.77
$\mathbf{I_p}$, plasma current (kA)	30	β_p, poloidal beta	1

About first exact solution, by using Eq. (12) for IR-T1 characteristics, we determined the four parameters a, b, α and β. The results are $= 7747.6976$, $b = 23.6462$, $\alpha = 6564.0172$, $\beta = 0.0054$ and because $a < 0$, then we choose the related poloidal magnetic flux function i.e. equations (8) and (10). For simplicity we consider only the terms with $k = 1,2$ and then by using the some points on the plasma boundary and also the value of the flux function ψ on the plasma boundary from an estimated equilibrium for IR-T1 tokamak [12], we determined the constants $D_{11} = 23.6145$, $D_{21} = 2.23151 \times 10^{-7}$, $D_{12} = 434.475$, $D_{22} = 2.7776 \times 10^{-7}$. Now we are ready to calculate the function $\Psi(r,z)$ for IR-T1 tokamak. The result is depicted in Fig. 2. As we can see, the equilibrium is not good (non tokamak-relevant equilibrium). Maybe this result is related to approximations that we considered in our calculation for IR-T1 tokamak, i.e. estimated equilibrium for this tokamak, and also maybe due to preference of this solution for a D-shaped and a toroidally diverted plasma not a circular cross section plasma.

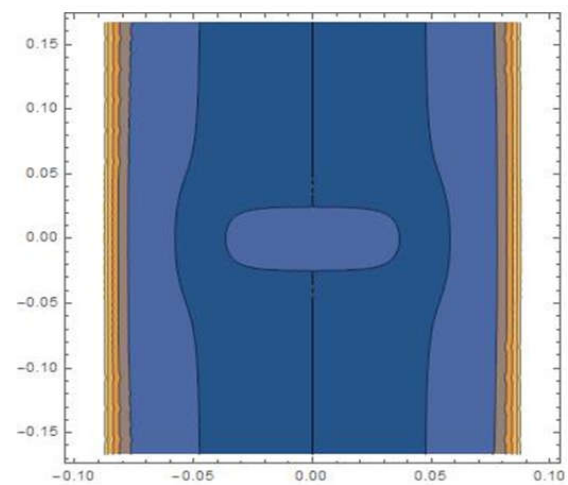

Fig. 2, Constant poloidal flux lines for IR-T1tokamak

In the second exact solution, by using Eq. (20) we determined the constants c_n for IR-T1 characteristics as:

$c_1 = 0.0365258 + (0.0879367)$, $c_2 = 0.132202 + (0.0462344)A$, $c_3 = 0.0744805 + (0.097145)A$

$c_4 = 0.0496243 \quad (0.0332743)A$, $c_5 = 0.001356 + (0.026839)A$, $c_6 = 0.002334 + (0.025819)A$

$$c_7 = 0.00187545 + (0.00429716)A \quad , \qquad (24)$$

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From equations (19), (22) and (23), we obtained $C_p = 1.695\,6$, $V \quad 0.05$ and $A = 3.9800813$ for IR-T1 characteristics. After obtaining the constants c_n and A, we calculate the function $\psi(r,z\ )$ for IR-T1 tokamak from Eq. (18). The result is presented in Fig. 3. We observe that the resulted equilibrium is good (tokamak-relevant equilibrium). It seems that this is due to the generality and high accuracy of this solution for all of the magnetic configurations of interest [11].

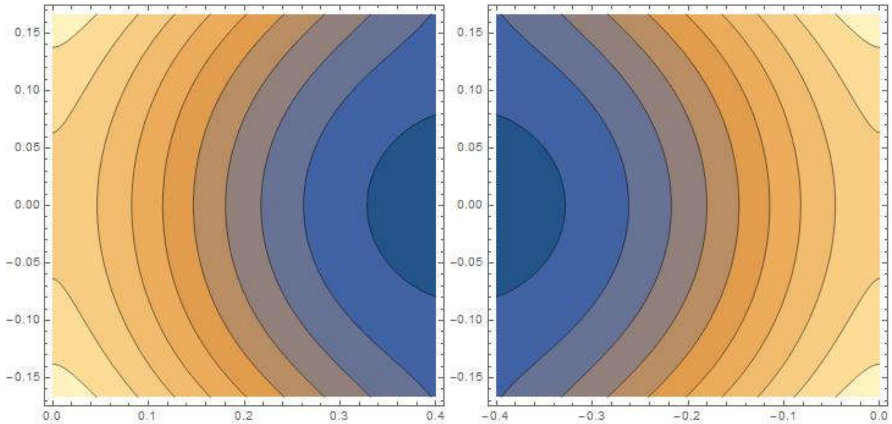

Fig. 3, IR-T1-like equilibrium; Left: positive radial coordinate half and Right: negative radial coordinate half

IV. DISCUSSION

The simplest useful mathematical model to describe equilibrium in fusion plasmas is achieved by combining Magnetohydrodynamic (MHD) equations with Maxwell's equations. The final result is the Grad-Shafranov (GS) equation. Here we presented the two families of analytical solutions. These solutions are applied to IR-T1 tokamak and poloidal magnetic flux is calculated. Due to the generality and high accuracy of the second exact solution for all of the magnetic configurations of interest, the result of this solution for IR-T1 tokamak is tokamak-relevant equilibrium compare to the first exact solution for this tokamak. We should notice that first these analytical solutions are used as benchmark of numerical equilibrium codes, and second provide a good model to test for stability without having to worry about accuracy and resolution issues arising from numerically computed equilibria.

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