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ORIGINAL RESEARCH

Feedback System Design for Plasma Horizontal Position Control in IR-T1 Tokamak

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Abstract Determination of accurate plasma horizontal position during plasma discharge and a control system based on feedback is essential to transport it to a safe position near the set point. The design of feedback controller is usually based on primitive modeling of the plasma itself. By using the plasma-circuits linearized model, proportional integral derivative based controllers and a model for the power supply of vertical coil system with some values for its poles, we analyzed step feedback response of the overall system of IR-T1 tokamak and corresponding Bode diagrams for two cases with and without the plasma resistance and the eddy currents distribution. Although experimental validation of the plasma model and also the overall system behavior is an important exercise, the results predicted in simulation can give us good insight about system.

Keywords Tokamak feedback control · Plasma-circuits linearized model · PID controller

Introduction

Plasma position, shape and current control is a matter of significant concern for the next generation of large tokamaks. Control of plasma displacement has an important role in magnetic confinement of plasma in tokamak system. Control of plasma current, position and shape in a tokamak is usually carried out on the basis of simple models [1–3]. There exist several non-linear plasma simulation codes that have been tested against present experiments [4, 5]. They provide simulation environments that can be used to check the performance of a control system. However, they are difficult to apply for routine use in controller design and performance analysis. This is mainly due to the long processing time needed for the simulation and the difficulty of deriving the linearized form usually needed for controller design. For this reason, the need for a simple but reliable plasma response model has become strong. Several linearized models have been proposed in the literature. Most of them are based on simplifying assumptions that dramatically reduce the dimensionality of the system [6, 7]. The models initially used for plasma control included more or less detailed circuit equations for both active and passive conductors, but treated the plasma in an approximate fashion, often as a set of filamentary coils with a prescribed, usually rigid, motion [1, 2]. The passive structures can also be schematized as a number of passive (short circuited) conductors. A typical assumption widely used in linear models is to neglect the plasma mass on the time-scale on which the control system works. This time-scale is much longer than the Alfven time and therefore the plasma can be assumed to evolve through a sequence of MHD equilibria, i.e. to be massless. This is the main basis for the perturbed equilibrium models, first introduced in [8] and later extended to include the effects of approximate flux conservation [9, 10]. A second and special in specific models assumption is that the toroidal plasma current density profile is assumed to be a function of only three parameters which are associated with specific physical quantities namely total plasma current, internal inductance and poloidal beta [11]. The perturbed equilibrium models have also been used with additional simplifying

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assumptions. The contribution of the eddy currents in the passive structures has often been disregarded. The accuracy of these models has generally been sufficient to develop the relatively simple but reliable Proportional Integral Derivative (PID) based controller for poloidal coils system. Finally a power supply (PS) model must be considered to complete the behavior of the overall system. A power supply is often highly non-linear but was approximated as a linear system. To model the power supply, we are using only a single pole filter approximating the time delay and bandwidth of the PS. Although this behavior of the overall system must be experimental validated but the results predicted in simulation can give us valuable insight about system. In this paper we consider the flat-top phase of IR-T1 tokamak discharge for plasma horizontal position control. In the low beta tokamaks as IR-T1, radial pressure balance is achieved by the poloidal field and toroidal force balance is achieved by the Lorentz force. But in the toroidal force balance problem, if the two opposite forces are not equal, then plasma intend to shift inward or outward which is dangerous for tokamak plasma. Determination of accurate plasma position during plasma discharge is essential to transport it to a control system based on feedback [12–31]. The paper has the following structure. In "Plasma-Circuits Linearized Model" section we present the plasma-circuits linearized model for application in plasma horizontal position control. Description of PID controller and PS of vertical coil system for IR-T1 tokamak will be presented in "PID Controller and Power Supply Design" section. In "Application to IR-T1 Tokamak and Related Results" section the step feedback response of the overall system of IR-T1 tokamak and corresponding Bode diagrams for two different cases with and without the plasma resistance and the eddy currents distribution are presented. "Discussion" section is devoted to discussion of the results.

Plasma-Circuits Linearized Model

The design of feedback controller is usually based on primitive modeling of the plasma itself. The validation of a plasma model is an important exercise but it also has a practical purpose in that it lays the ground for model based controller design. The linearized response of the plasma includes describing the general response of the plasma to changes in the currents in the passive structures of the vessel and to changes in the currents in the active poloidal field (PF) coils. This plasma response will depend on the particular plasma equilibrium and, of course, on the tokamak geometry. Because plasma motion should be controlled in the region near equilibrium $(\boldsymbol{x}_0, \boldsymbol{u}_0)$, plasma dynamics can be approximated by a linear time invariant state equation. Then we are mainly interested in the flat-top phase control, which is characterized by a rather slow dynamics (approximately static). The time evolution of the plasma response is determined by Ohm's law combined with MHD equilibrium and by all the additional circuit equations:

$$\frac{d\boldsymbol{\Psi}}{dt} + \boldsymbol{RI} = \boldsymbol{V} \tag{1}$$

where $\boldsymbol{I}$ is the set of currents flowing in the external (active and passive) conductors, $\boldsymbol{\Psi}$ is the set of fluxes linking these currents,$\boldsymbol{R}$ is the resistance matrix of the circuits and $\boldsymbol{V}$ is the complete set of applied voltages, with $\boldsymbol{V} = 0$ for the passive circuits. The derivation of linearized model in the absence of any external disturbances (when we are in the controller design step) is now straightforward:

$$\boldsymbol{L}^* \frac{d\boldsymbol{x}}{dt} + \boldsymbol{Rx} = \boldsymbol{u} \tag{2}$$

In which the internal state vector is $\boldsymbol{x} = \left[\delta\boldsymbol{I}, \delta\boldsymbol{I}_p\right]^T$ and the input vector is $\boldsymbol{u} = [\delta\boldsymbol{V}, 0]^T$. The quantities $\delta\boldsymbol{I}, \delta\boldsymbol{I}_p$ and $\delta\boldsymbol{V}$ represent linearized deviations about their nominal values. The matrix $\boldsymbol{L}^* = \frac{\partial\left[\Psi, \Psi_p\right]^T}{\partial\left[I, I_p\right]^T}$ is the modified inductance matrix. Equation (2) can be converted to the standard state-space representation used widely in control theory:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{Ax} + \boldsymbol{Bu} \tag{3}$$

where the matrix determining the poles of the system is $\boldsymbol{A} = -\boldsymbol{L}^{*-1}\boldsymbol{R}$ and the matrix describing the coupling between the applied voltages and the internal states is $\boldsymbol{B} = \boldsymbol{L}^{*-1}$. The linearized model can also predict linearized output parameters $\boldsymbol{y}$ other than the state variables themselves, such as field or flux measurements, separatix gap deviations or velocity for diverted plasmas, using the standard output equation:

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du} \tag{4}$$

where $\boldsymbol{C}$ and $\boldsymbol{D}$ are the state-to-output and input-to-output matrices, respectively. The presence of possible β_p and l_i disturbances can be taken into account by introducing two additional matrices $\boldsymbol{E}$ and $\boldsymbol{F}$ such that:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{Ax} + \boldsymbol{Bu} + \boldsymbol{E}\frac{d\boldsymbol{\omega}}{dt}, \boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du} + \boldsymbol{F\omega} \tag{5}$$

where $\boldsymbol{\omega} = \left[\delta\beta_p, \delta l_i\right]^T$. These matrices $\boldsymbol{E}$ and $\boldsymbol{F}$ are determined using the system response to canonical disturbances. The $\boldsymbol{E}$ and $\boldsymbol{F}$ matrices are not used in this present paper since we shall only be considering external actions on the plasma via the coil voltage vector [11].

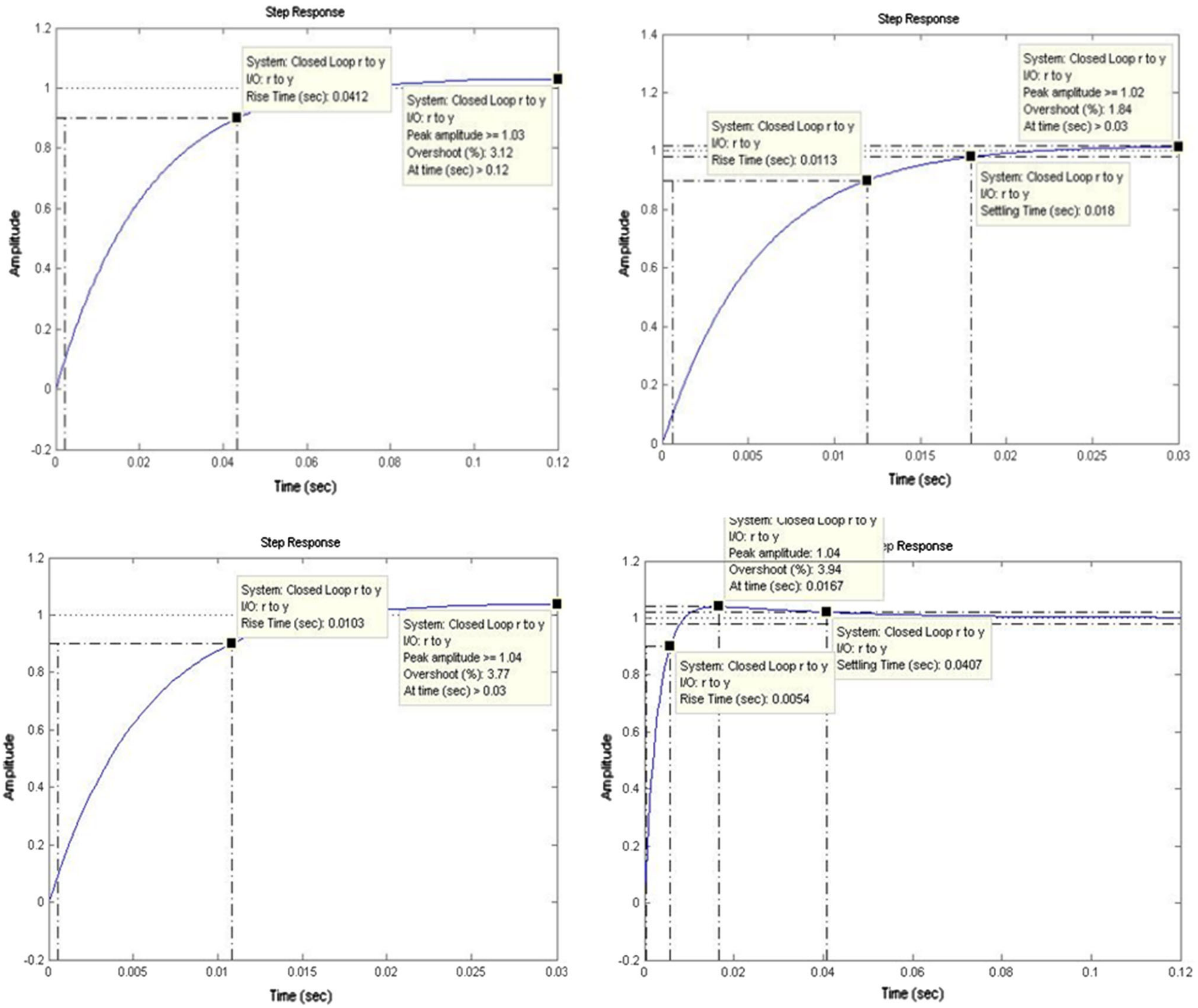

Fig. 1 Step closed loop response of the overall system of IR-T1 tokamak without the plasma resistance and the eddy currents distribution; *top left* $a = 1$, *top right* $a = 2$, *bottom left* $a = 5$ and *bottom right* $a = 10$

PID Controller and Power Supply Design

From the full order model that was presented in Sect. 2, a reduced order model with one input and one output was extracted and based on that a Proportional Integral Derivative (PID) based controller for vertical coil system of IR-T1 tokamak was designed. In particular, the gain and phase margins were made sufficiently high in order to guarantee the stabilization of a number of models suitably chosen in the assumed working envelope. Because of on-line computing requirements, the controller is constrained by the shortness of the control intervals. This factor requires the control model to be simple and preferably linear. We choose the following transfer function of PID controller for horizontal plasma position control (controller with filtered derivative) [13]:

$$G_{PID}(s) = K\left[1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right] \tag{6}$$

where K is the gain; T_i is the integral time; T_d is a time constant used to approximate the derivative action and $N = 100$. To model the power supply of vertical coil system of IR-T1 tokamak, we are using only a single pole filter, approximating the time delay and bandwidth of the PS. The proposed transfer function for PS system is:

$$G_v(s) \cong K_v \frac{e^{-T_v s}}{s + a} \tag{7}$$

where $K_v \simeq 1$ and $T_v = 1$ µs and a represent the pole of vertical coil system.

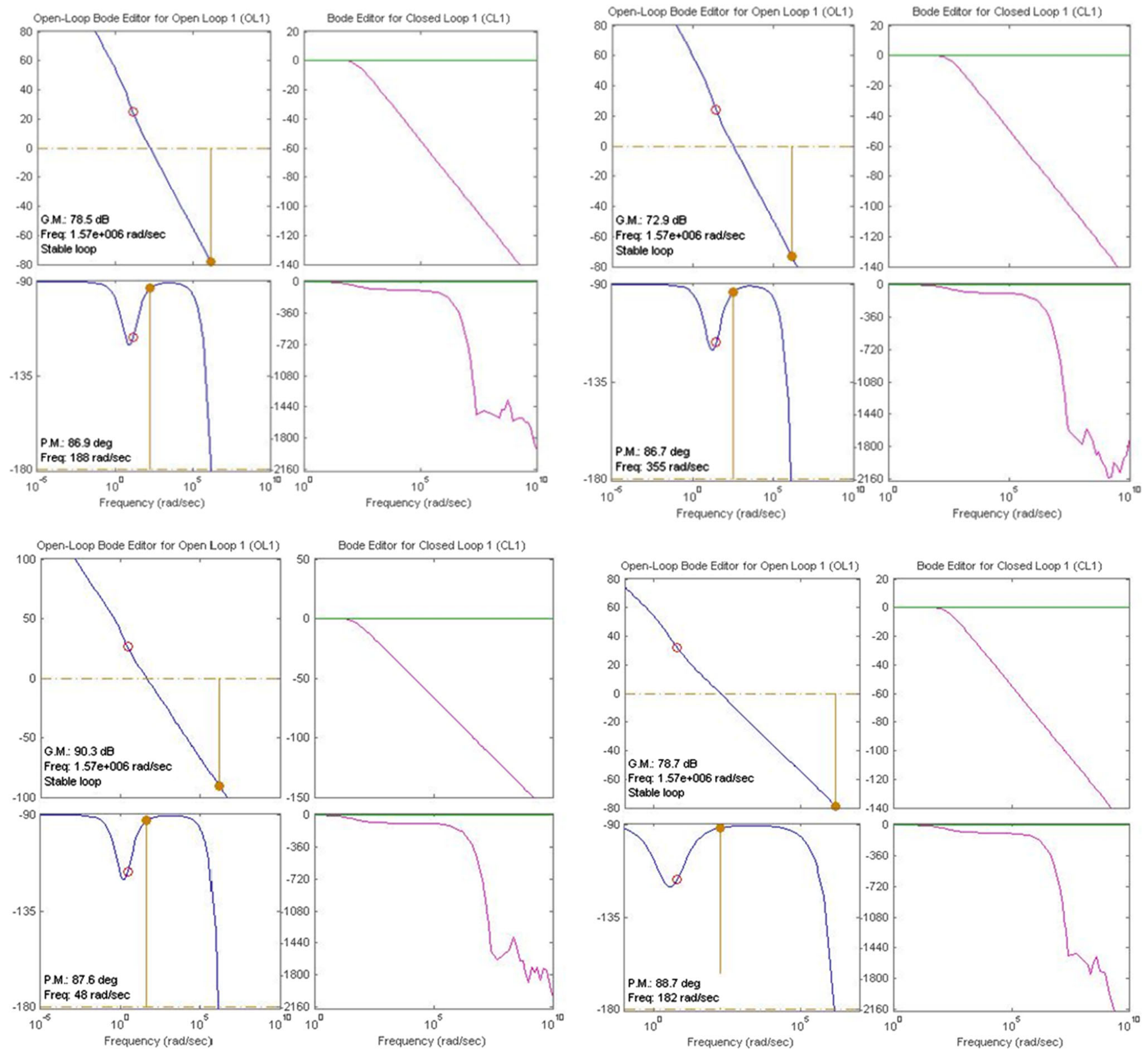

Fig. 2 Open loop and closed loop Bode diagram of the overall system of IR-T1 tokamak without the plasma resistance and the eddy currents distribution; *top left* $a = 5$, *top right* $a = 10$, *bottom left* $a = 1$ and *bottom right* $a = 2$

Application to IR-T1 Tokamak and Related Results

With considering of Eqs. (3) and (4) for plasma horizontal position controller design problem, the values of A, B, C and D for IR-T1 characteristics can be calculated by using the method proposed in ref [14]. With these values we can derive transfer function for the plasma response of IR-T1 tokamak. The transfer function of PID controller and PS system for this tokamak are described in Sect. 3. The transfer function of whole system is product of these three transfer functions. The blocks described above put together and a closed-loop simulation will be carried out. In this paper we studied two cases. In first case we neglect the plasma resistance and the eddy currents distribution. If the plasma motion in an equilibrium state is only slightly affected by eddy currents, eddy current dynamics may be ignored in the state equation. We have model with linearized deviation of vertical coil current from equilibrium value as the state vector and change in vertical field voltage as the input. Here we have a voltage driven model [15]. For obtaining a good insight about system, first we should study system stability then we deal with step change i.e. a unit value change in vertical field voltage as input and then

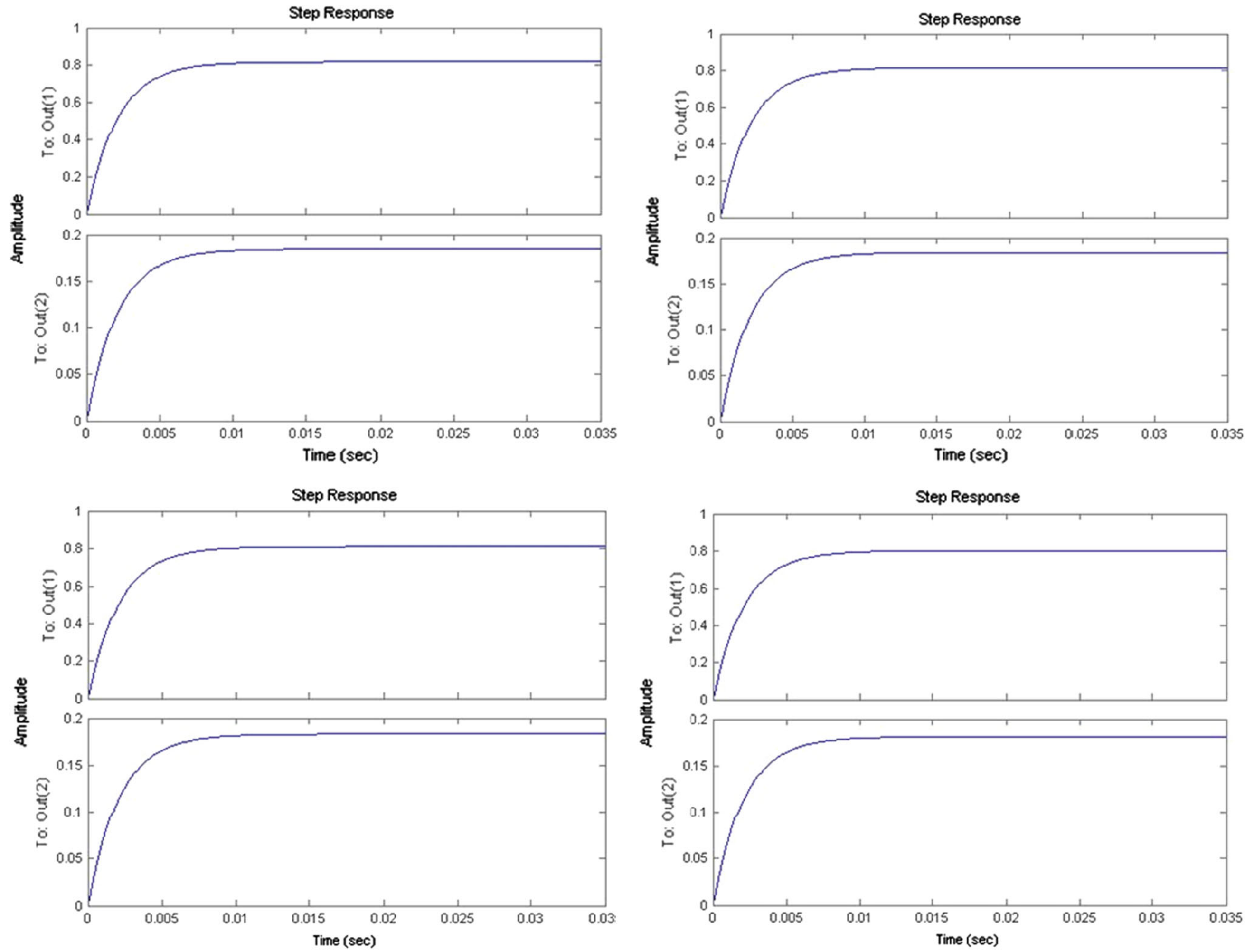

Fig. 3 Step closed loop response of the overall system of IR-T1 tokamak with the plasma resistance and the eddy currents distribution; *top left* $a = 1$, *top right* $a = 2$, *bottom left* $a = 5$ and *bottom right* $a = 10$

studied feedback response of the system. In Fig. 1 we present step closed-loop response of the overall system of IR-T1 tokamak for first case. Actually we choose four different values for a of the PS system to find optimum value of that. To guarantee the stabilization of model, in Fig. 2 we show Bode diagrams of the response (open loop and closed loop response). We have gain in top of each figure of Fig. 2 and phase in bottom of it. Also we have gain margin (G.M.), corresponding frequency and loop stability into top figures and phase margin (P.M.) and corresponding frequency into bottom figures of Fig. 2. From Figs. 1 and 2, we can see that $a = 2$ has approximately better performance in view of control and stability than other responses and in which gain and phase margins are sufficiently high. This value give us an estimate and actually the optimum value of a must be experimentally validated.

In second case we consider these effects (plasma resistance and the eddy currents distribution). We have linearized deviation of vertical coil current from equilibrium value as input and two new parameters that are related to plasma and eddy currents as state vector. We called this model as current driven model [15]. The time constant of eddy current responses which is generally considered to be the skin time of the vacuum chamber is very short compared to time constant of variation of vertical field coil current and the eddy currents become almost stationary after the skin time. The uniform eddy current is sustained during the time constant of variation of vertical field coil current. In the case of control with eddy currents effect, the sampling time of the plasma position controller should be shorter than the time constant of the eddy current. Same procedures are done for second case in Figs. 3 and 4. From these figures, we can see that there is no significant difference between the values of a. Again this issue must be experimentally validated and in future work we will study the experimental results against simulation predictions.

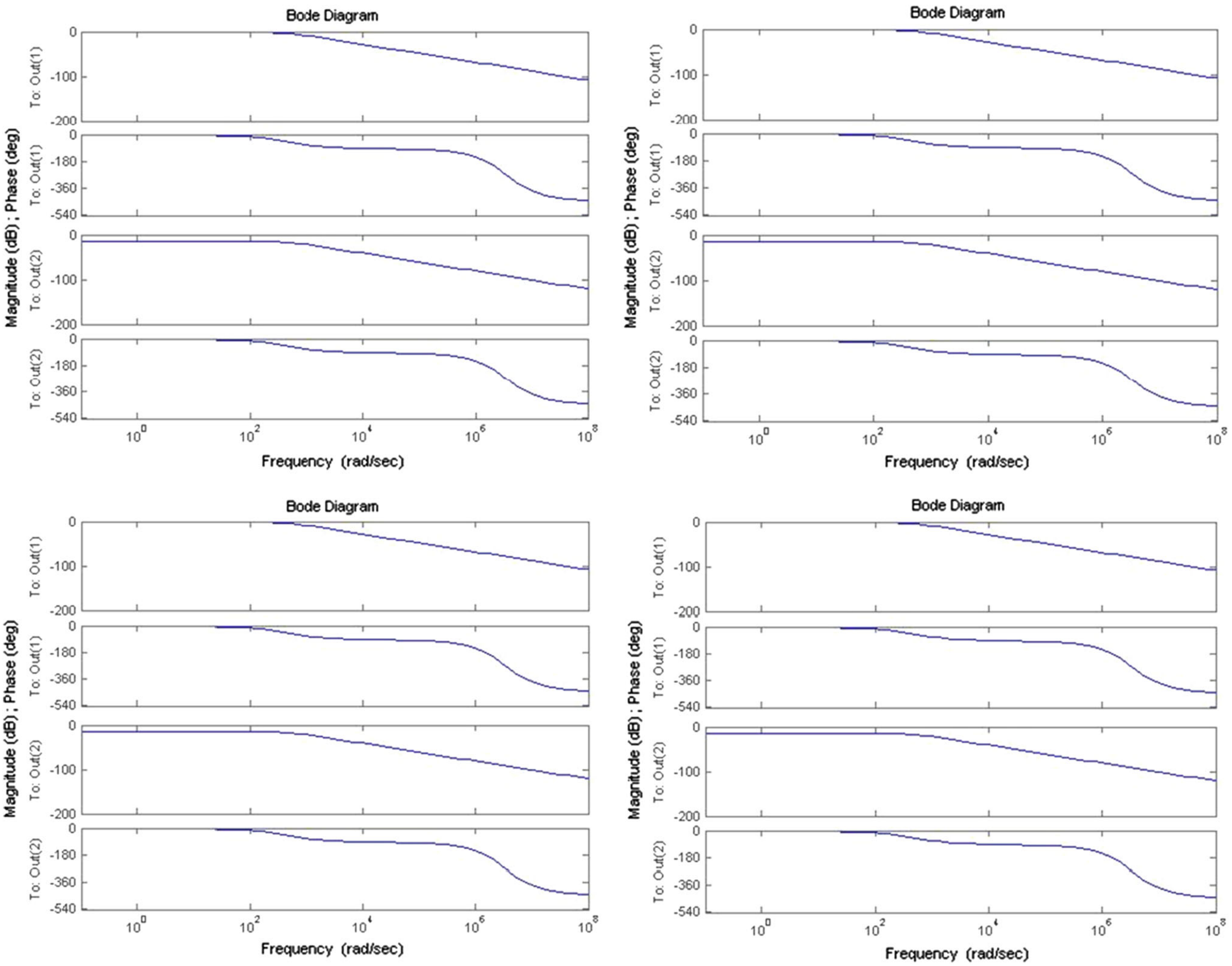

Fig. 4 Closed loop Bode diagram of the overall system of IR-T1 tokamak with the plasma resistance and the eddy currents distribution; *top left* $a = 1$, *top right* $a = 2$, *bottom left* $a = 5$ and *bottom right* $a = 10$

Discussion

Control of plasma horizontal displacement has an important role in magnetic confinement of plasma in tokamak system. First step for control of plasma horizontal position is the controller design. The design of feedback controller is usually based on primitive modeling of the plasma itself. The validation of a plasma model is an important exercise but it also has a practical purpose in that it lays the ground for model based controller design. In this paper we have worked with the plasma-circuits linearized model as a base for controller design. Also we proposed PID controller and PS model of vertical coil system for IR-T1 tokamak. Step closed-loop response of the overall system of IR-T1 tokamak and corresponding Bode diagrams for two cases with and without the plasma resistance and the eddy currents distribution are presented. The predicted results must be experimentally validated and in future work we will study the experimental results against simulation predictions.

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