Scaling properties of one-dimensional off-diagonal disorder

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Validity of the single parameter scaling (SPS) in one dimensional Anderson model with purely off-diagonal disorder is being studied. It is shown that the localized region with standard symmetry is divided into two regimes: SPS and non-SPS. Scaling relations of the Lyapunov Exponent are proposed for these two regimes. In the non-SPS regime, in additional to the localization length, there exists a new length scale which is related to the integrated density of states. A physical interpretation of the new length is the cross-over length which separates regions with chiral symmetry from those that have standard symmetry.

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I. INTRODUCTION

It is well-known for about forty years that all electron states in standard one dimensional (1D) disordered models are localized for any strength of disorder, and there is no localization transition in 1D systems [1]. However, in the case of off-diagonal disorder, there is an anomalous localized state at the band center [2,3,4,5]. It has been proposed that the Lyapunov Exponent (L.E.) is the appropriate scaling variable to describe fluctuations of the conductivity.

$$\gamma(N) = \frac{1}{2N}\ln(1+\frac{1}{g}) = -\frac{1}{2N}\ln(T) \qquad (1)$$

where $g(=T/R)$ and T are conductance and transmission coefficients through the system with length N.

A reason for the revival of the interest in 1D disordered model is due to the revision of the well-known Single Parameter Scaling (SPS) hypothesis. According to this hypothesis [6], there exists a single parameter, conductance g, which determines scaling properties of $g(N)$. Soon after the report of SPS, it became clear that one should consider scaling of the full probability distribution of conductivity.

In order to take fluctuations of conductance into account, one should consider a parameter γ (L.E.) defined in Eq.(1) instead of conductance g itself. In the thermodynamic limit, $\gamma_0 = \gamma(N \to \infty)$ has a non-random value which is the inverse of the localization length λ. This parameter has normal distribution for $N \gg \lambda$, and its dispersion σ_γ obeys the law of large numbers and is not an independent variable.

$$\frac{1}{\tau} = \frac{\gamma_0}{N\sigma_\gamma^2} = 1 \qquad (2)$$

where the parameter τ is usually defined as a conventional scaling parameter in literature. The above equation was originally derived by Anderson *et al.*[7] by using the random phase hypothesis.

However, as shown in [8] without the assumption of phase randomization, 1D SPS (Eq.[2]) is violated, where states are much far apart from each other than the localization length (λ).

This new characteristic length scale ℓ_s which is related to the distance between states, for the states near the band center is defined in terms of total number of states ($N(E)$) whose energy is less than E [10]. This criterion was initially extracted from the exact calculation of the variance of L.E. for the Anderson model with Cauchy distribution of the site energies [8]. However in contradiction with violation of SPS in 1D [11], it has been recently shown using exact diagonalization [12] and transfer matrix method [13] that at special point $E = 0$, SPS holds perfectly in 1D.

In 2D case, the interest is motivated by the experimental observations of a metal-insulator transition which is at odds with the SPS for noninteracting electrons [14]. The validity of SPS in 2D is currently very controversial. There exists some numerical analysis of 2D Anderson model which confirms the SPS hypothesis [15,16,17]. Other studies suggest a two-parameter scaling [12,13,18]. It has been shown [18] that 2D SPS does not follow Eq.(2).

In 1D systems with off-diagonal disorder (random hopping model), it is clear that an anomalously localized state at $E = 0$, results in a violation of SPS. Divergence of the localization length and density of states at the band center in this model [19,20], is in contradiction with the scaling theory. Unusual properties of this model are due to chiral symmetry [21,22]. In an interval close to an anomalous state, SPS does not hold [10]. The main objective of the present paper, is to answer how far from the anomalous state (at $E = 0$), 1D SPS will again be held.

In this paper, with care of some debates on the validity of 1D SPS, we reexamine the scaling properties of one-dimensional system with purely off-diagonal disorder by using transfer matrix method. Our attention is on a region near the band center which contains strongly localized states with standard symmetry. In the strong

localization limit, it will be shown that the L.E. distribution function is normal. In this region, it is shown that there exists a new length scale which is the same as the length scale (ℓ_s) defined in Eq.(11) [8]. The SPS exists as long as the localization length λ exceeds ℓ_s. In the SPS region, it is shown that the L.E. only depends on the disorder strength as $\gamma_0 \propto \sigma^2_{\ln(t)}$. The scaling properties of the non-SPS region which has been reported in Ref. [24], is confirmed by a data collapse. The variance and mean of the L.E. for different disorder strengths, system sizes and also for various range of energy spectrum, lie on a single curve when they are expressed in terms of the scaling parameter (τ) defined in Eq.(2) as a function of the ratio $\kappa = \lambda/\ell_s$. It can also provide a physical interpretation for ℓ_s as a cross-over length between two chiral and standard symmetries.

This article is organized as follows: Section **II** describes our model and the exact calculation of all L.E. moments at the band center. Section **III** describes the transition from chiral symmetry region to localized region by the calculation of L.E. distribution function and its mean. In this Section, it will be shown that the localized region is divided into non-SPS and SPS regimes. A new length scale(ℓ_s) which controls the scaling theory is defined in Section (IV). We try to find a meaningful physical interpretation for the new length scale as a cross-over length in Section (**V**). Discussions and conclusions are finally presented in Section **VI**.

II. MODEL AND MOMENTS OF LYAPUNOV EXPONENT

We consider non-interacting electrons in 1D disordered systems within a tight binding approximation. The Schroedinger equation with the assumption of nearest-neighbor hopping becomes

$$\varepsilon_i\psi_i + t_{i,i+1}\psi_{i+1} + t_{i-1,i}\psi_{i-1} = E\psi_i \qquad (3)$$

where E is the energy corresponding to the electron wave function. $|\psi_i|^2$ is the probability of finding the electron at site i, ε_i are the site potentials and $t_{i-1,i} = t_{i,i-1} = t_i$ the hopping terms. Using the transfer matrix method, one can relate the electron wave functions at the two ends of the system to each other. In our model, we consider all site energies to be zero and a periodic boundary condition on hopping terms as $t_1 = t_{N+1}$. All energies which appear, are scaled by typical mean of hoppings terms t_0, where $\ln(t_0) =< \ln(t_i) >_{c.a.}$. Here, **c.a.** refers to the configurational average. The L.E. can be extracted from the eigenvalues of the total transfer matrix [24].

As proved in Ref.[24], the L.E. at $E = 0$ has a *semi-Gaussian* distribution with a mean which can be derived in terms of the pair correlation function. By having the distribution function, higher powers of the L.E. can be simply derived in the case of correlated and uncorrelated disorder at the band center ($E = 0$) as:

$$< \gamma^2 >= \frac{\pi}{2} < \gamma >^2; < \gamma^3 >= \pi < \gamma >^3; ...;$$
$$< \gamma^n > \propto < \gamma >^n \qquad (4)$$

Therefore, higher moments of the L.E. can be written as:

$$\sigma^2_\gamma =< (\gamma - < \gamma >)^2 >= (\frac{\pi}{2} - 1) < \gamma >^2$$
$$< (\gamma - < \gamma >)^3 >= (2 - \frac{\pi}{2}) < \gamma >^3 \qquad (5)$$

This can be generalized to the n'th moment of the L.E.

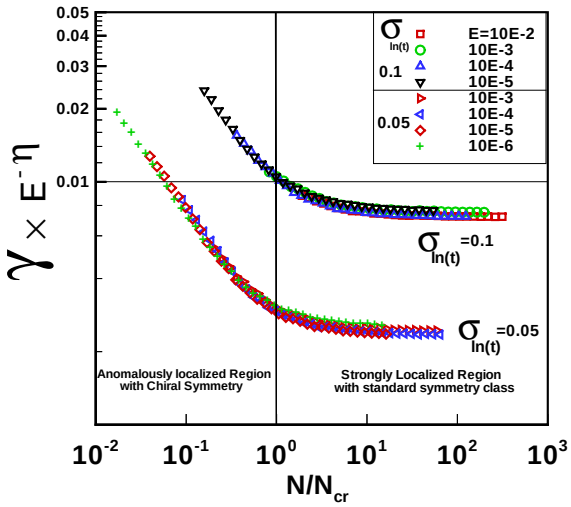

FIG. 1: The size dependency of the Lyapunov Exponent for different energies with $\eta = 0.18$.

which will be as $< (\gamma - < \gamma >)^n > \propto < \gamma >^n$. By paying our attention to the L.E. form at $E = 0$ ($< \gamma > \propto \sigma_{\ln(t)}/N^{1/2}$), it can be seen that the variance of the L.E., scales according to the law of large numbers for uncorrelated disorder. As it has been mentioned by Anderson. et al [7], the localization properties can be described by a variable (such as L.E.) whose width of distribution function follows the law of large numbers.

$$\sigma^2_\gamma = (1 - \frac{2}{\pi})\frac{\sigma^2_{\ln(t)}}{N} \qquad (6)$$

This equation and its equivalence in Eq.[5] are consistent with the result of Ref.[22], where it was derived for a weak disorder by solving the Fokker-Planck equation. However, the size dependence of the L.E. variance will change when the disorder becomes correlated. As a result, the L.E. distribution function and all its higher moments converge for large system sizes. So, L.E. is a good variable to describe statistical properties of disordered systems.

III. SCALING AND DISTRIBUTION FUNCTION OF LYAPUNOV EXPONENT

A. Scaling of Lyapunov Exponent

We calculate the mean and variance of L.E. by using the transfer matrix method when randomness is imposed on $\ln(t)$'s. The study of L.E. close to the band center results in the coexistence of two symmetries in this system. It can be shown that there is a chiral symmetry at $E = 0$. This is a significant property of purely off-diagonal disorder with nearest-neighbor approximation. However, at energies close to the band center, and for lengths greater than a cross-over length (N_{cr}), chiral symmetry is broken. At sufficiently long lengths, localization properties will flow to those of the standard symmetry class. All states in this regime are strongly localized.

For any realization of the disorder, the energy density of states is symmetric around the band center. This symmetry, which originates from the fact that the disorder preserves the bipartite structure of the lattice, is referred to as chiral symmetry. The chiral symmetry is broken by, e.g., on-site randomness or next-nearest-neighbor hopping [4,22].

In the case of purely onsite disorder case which was originally considered by Anderson [1], one distinguishes three universality classes, corresponding to the presence or absence of time reversal and spin-rotation symmetry. These three classes are called orthogonal, unitary, and symplectic [22]. Here, we will refer to these as the three standard universality classes. In this paper, it will be shown that the standard symmetry region (strongly localized region) is also divided into two regimes (non-SPS and SPS regimes defined in section (III.C) and Fig.(3)), which depend on the number of scaling parameters.

Fig.(1) shows the scaling properties of L.E. near the band center. All data with various energies lie on a single curve when $(\gamma \times E^{-\eta})$ is plotted in terms of the dimensionless variable (N/N_{cr}). It confirms a power law divergence of the localization length where energies belong to the non-SPS regime (Fig.(3)). Therefore, the following scaling law of L.E. at $E \neq 0$ can be proposed [24].

$$(\gamma \times E^{-\eta}) \propto \begin{cases} \sigma^2_{ln(t)} (\frac{N}{N_{cr}})^{-1/2} & N/N_{cr} \ll 1 \\ \sigma^2_{ln(t)} & N/N_{cr} \gg 1 \end{cases} \quad (7)$$

where the cross-over length is as

$$N_{cr} \propto E^{-2\eta} / \sigma^2_{ln(t)} \quad (8)$$

and $\eta \approx 0.18 \pm 0.03$. However, for energies very close to the band center, η has a small energy dependence. Fig.(1) shows a transition from the region with chiral symmetry ($N \ll N_{cr}$) to the region with standard symmetry ($N \gg N_{cr}$). It was also checked that each of the data sets do not collapse on each other when one uses the logarithmic energy dependence of the localization length as seen in Refs.(3,5).

B. Distribution Function of Lyapunov Exponent

The localization properties of different symmetry regions can be also characterized by the distribution function of L.E. As it was mentioned in section (II), at the band center, the distribution of L.E. is semi-Gaussian. For zero energy ($E = 0$), numerical evidence in Fig.(2.a) confirms such analytical distribution function for all system sizes.

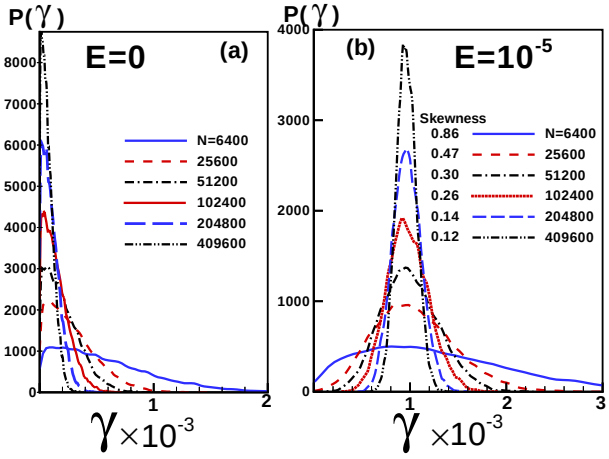

FIG. 2: Lyapunov Exponent Distribution Function for different system sizes at a) $E = 0$ and b) $E = 10^{-5}$. Disorder strength is considered to be 0.1. The number of samples is 2×10^4 configurations.

For energies near the band center ($E \neq 0$), and for system sizes smaller than the cross-over length ($N \ll N_{cr}$), the distribution of L.E. is *semi-Gaussian*. In this region, the band center behavior is dominant. However, as the system size increases, the distribution function becomes more *Gaussian*-like for $N \gg N_{cr}$. This region has a regular Anderson-like behavior with standard symmetry class. Fig.(2b) shows distribution function of the L.E. for the same system sizes as the band center case. It can be seen that there exists again a transition between these two symmetries; from chiral to standard symmetry or from semi-Gaussian to Gaussian distribution. For the sake of completeness, the skewness of the distribution functions has been calculated as a measure of the symmetry of the distribution. It can be defined as [25]:

$$\text{Skewness} = \frac{< (\gamma - \overline{\gamma})^3 >}{< (\gamma - \overline{\gamma})^2 >^{3/2}} \quad (9)$$

Distribution close to the normal form, has a skewness equal to zero. A distribution whose skewness has absolute value less than 0.5 is considered fairly symmetrical. Therefore, distributions of long enough systems in Fig.(2b), are very close to the Gaussian form. In the SPS regime (Fig.(3)) where the L.E. is independent of size and energy, distribution function of L.E. is exactly

Gaussian (the skewness order of 10^{-3}) and independent of size. All distribution curves have been softened by the Kernel smoothing method [26] without changing any statistical characteristic of distributions.

C. SPS and Non-SPS Regimes

For a fixed length (N), Eq.(7) proposes a critical energy point which separates two regions with different symmetries. In fact, for energies greater than $\varepsilon_{cr.}^{(1)} \propto (N\sigma^2_{\ln(t)})^{\frac{-1}{2\eta}}$, the system is in the localized region. Fig.(3) shows the energy dependence of L.E. in the localized region. It can be seen that there is a second critical point ($\varepsilon_{cr.}^{(2)} \approx 10^{-2}$ in Fig.(3)) where the energy spectrum is divided into SPS and non-SPS regimes. In the SPS regime, L.E. is independent of energy and the scaling theory is valid (Eq.(2)). Although, near the band edges anomaly, L.E. will become energy dependent. In the SPS regime, it has been checked (Figs.(1,3)) that in contradiction to the result of Ref.[27], the L.E. only depends on disorder strength. Fig.(4) shows that the L.E. is proportional to the square of disorder strength.

$$\gamma_0 \propto \sigma^2_{\ln(t)} \tag{10}$$

The line Fitted on data in Fig.(4) has a slope equal to the value 2. The coefficient of the above scaling law is $(\frac{1}{3.00\pm0.2})$ which can be extracted for a fixed disorder strength ($\sigma_{\ln(t)} = 0.1$) and for an energy ($\varepsilon = 0.1 \geq \varepsilon_{cr.}^{(2)}$) in the SPS regime. The skewness of this point is about 0.005. This skewness shows an exactly Gaussian form for L.E. in this regime.

It can be seen that matching of two Eqs.(7,10) at the boundary of non-SPS to SPS regime ($E = \varepsilon_{cr}^{(2)}$) leads to a second critical point ($\varepsilon_{cr}^{(2)}$) which is independent of all system parameters (a constant).

Now, we numerically study the variance of L.E. as a function of system parameters. Fig(3) shows $N\sigma^2_\gamma$ versus energy at fixed system size. As it can be seen, the variance of L.E. is approximately independent of energy at energies in the SPS and also non-SPS regimes. Figs.(3,4) show that size and disorder strength dependence of the variance of L.E. follows from Eq.(6). As it is clear from a fitted line (with slope 2) on data in Fig.(4), the quantity $N\sigma^2_\gamma$ is proportional to $\sigma^2_{\ln(t)}$. The size dependence of the L.E. variance ($\sigma^2_\gamma(N)$) is shown in the inset Fig.(4). The L.E. variance decreases with the inverse of the system size similar to the size dependence of variance at the band center (Eq.(6)). So, the variance of L.E. at the band center can be generalized to other energies near the band center.

IV. VIOLATION OF SINGLE PARAMETER SCALING

According to Eq.(2), the two parameters of the distribution reduce to only one. Parameter τ can be defined as a measure of SPS in that equation. Now, in this Section, we try to find the independent parameters of the system in non-SPS regime. It is again stressed that the random hopping model away from the $E = 0$ and in the localized region is being studied. The quantity of interest is ℓ_s that is related to the integral density of states from the

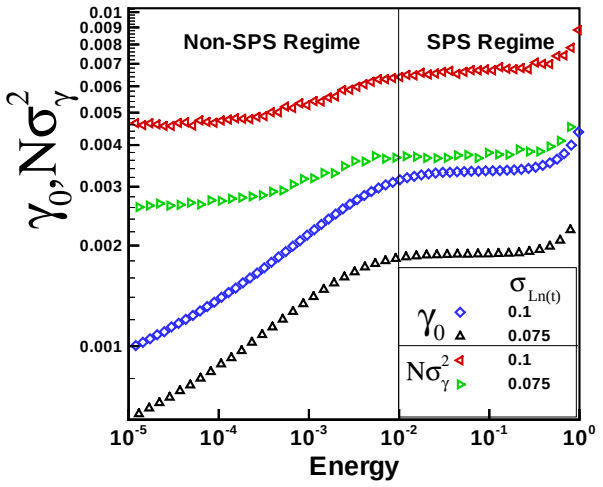

FIG. 3: SPS to non-SPS transition with plotting Lyapunov Exponent and $N\sigma^2_\gamma$ versus energy for system size 10^5. $\varepsilon_{cr.}^{(1)}$ which depends on the disorder strength, is about 5×10^{-9} and 3×10^{-7} for $\sigma_{\ln(t)} = 0.1$ and 0.075 , respectively. $\varepsilon_{cr.}^{(2)}$ is equal to 10^{-2}.

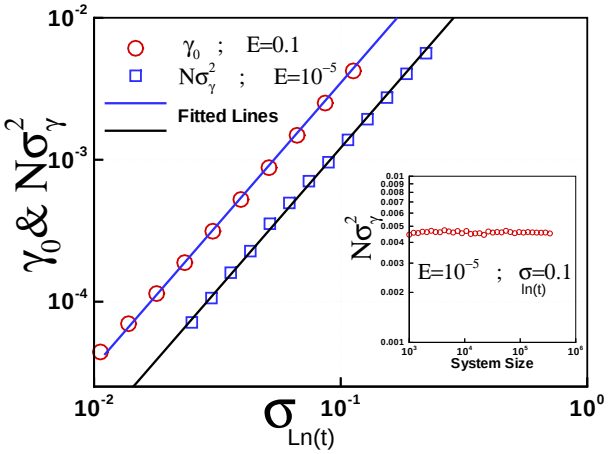

FIG. 4: Standard deviation of the Lyapunov Exponent versus disorder strength for system with length 10^5. Dependency of γ_0 to disorder strength in the SPS regime ($E = 0.1$). Inset figure shows the system size dependence of the variance of Lyapunov Exponent.

band center to a given energy, normalized by the total number of states in the band.

$$\ell_s = \frac{1}{\sin(\pi N(E))} \tag{11}$$

For the Anderson model, $N(E)$ can be computed by the node-counting theorem [29]. By starting an initial vector in transfer matrix method, we count the number of wave function nodes as the length is scanned.

Fig.(5) shows the numerical result of inverse scaling parameter ($\tau(\kappa)$) in terms of the dimensionless parameter ($\kappa = \frac{\lambda}{\ell_s}$) for different values of disorder strength and energy.

The data included in this graph correspond to the localized regime with standard symmetry class where the L.E. has a Gaussian distribution. The inset Fig.(5) shows that the skewness of the data are less than 0.25 for $\kappa > 0.1$. All data are in the region $N \gg \ell_s$. What is important, is that all data with different values of energy and disorder strength and also system size collapse to a single curve when they are expressed in terms of $1/\tau$ and κ. Therefore, for $\kappa \ll 1$ (non-SPS regime) variance of L.E. depends on two parameters; κ and the mean of L.E. In the case of $\kappa \gg 1$, the inverse of the scaling parameter $(1/\tau)$ in the present model goes to the value 0.5 which is different from unity. Therefore, our expression from non-SPS is only the deviation of Eq.(2) from 0.5. Energies are

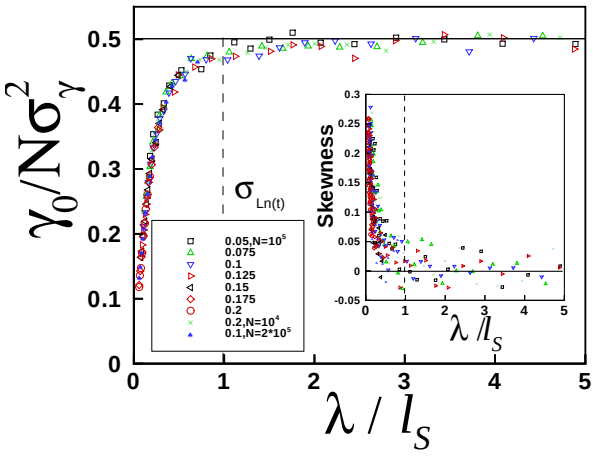

FIG. 5: Measure of the validity of the Single Parameter Scaling by inverse Scaling Parameter ($1/\tau = \gamma_0/(N\sigma_\gamma^2)$) in terms of $\kappa = \lambda/\ell_s$ for disorder strengths from $\sigma_{\ln(t)} = 0.05$ to 0.2. Energies depending on disorder strength, are scanned from $E = 10^{-7} - 10^{0}$. Here $N \gg \ell_s$. Inset figure shows the skewness of the data. The number of samples is 10^4 configurations.

scanned from near the band center to the middle of the conduction band, and far from the second anomaly in the band edges. In fact, close to any anomaly (here a delocaliztion at the band center), a violation of SPS will happen [10] in an interval close to the anomalous state. It can be seen that for $\kappa \ll 1$, SPS is clearly violated (non-SPS region). For $\kappa \gg 1$, independent of disorder strength, standard SPS is restored again. In the SPS spectral region, the localization length is a macroscopic length as only one parameter of system. The conductance can be defined as a function of this variable.

Both length scales ℓ_s and the localization length are decreased when energy is swept from the vicinity of the band center to the band edges. In energies close to zero, the new length scale is greater than the localization length. There is a critical energy where both length scales are of the same order ($\lambda \approx \ell_s$). Far from the band center, the localization length is independent of energy and the new length scale steeply decreases so that it would be much smaller than a macroscopic localization length.

This result confirms the general conjecture of the authors [8] that the second moment of the distribution function of L.E. can be universally described in terms of variables τ and κ regardless of the microscopic nature of the models under consideration. The form of the function $\tau(\kappa)$ may differ for different models and its essential behavior is not universal. All models follow $\tau(\kappa) = 1$ for $\kappa \gg 1$, while in the hopping disorder model, it is $\tau(\kappa) = 2$. In the model studied in the present paper, for $\kappa \ll 1$, it can be seen an exceptional behavior compared to other models such as Lloyd model [8,10,23] and Anderson (onsite disorder) and superlattice models [23]. The scaling parameter τ increases with κ in the hopping disorder model for $\kappa \ll 1$, while in the above models, τ steeply decreases with κ. As an example, analytical calculations carried out in Ref.[8] for the Lloyd Model produced $\tau = (\pi/2)\kappa$.

The power law form is the best fitted curve for $\kappa \ll 1$.

$$\frac{1}{\tau} \propto \alpha\kappa^{\beta} \tag{12}$$

Coefficients are estimated by using a linear regression in log-log plot. The fitted power of κ and its coefficient are $\beta = 0.815 \pm 0.008$ and $\alpha = 1.64 \pm 0.03$, respectively. Since there is a kind of delocalization at the band center, the L.E. sharply decreases near the band center compared to $N\sigma_\gamma^2$ (Fig.(3)). Therefore, the inverse scaling parameter decreases in energies close to the band center.

V. CROSS-OVER LENGTH AS A PHYSICAL INTERPRETATION OF ℓ_s

Numerical results in Fig.(3) show that as a crude approximation, one can consider the variance of L.E. independent of energy in the non-SPS and SPS regimes (not near their transition point). It was shown that the form of variance near the band center is similar to its form in the band center (Eq.(6)). By using this form of the variance and the scaling relations derived in Eqs.(7,10), the scaling parameter function (τ) can be proposed in the non-SPS and SPS regimes.

First, we investigate the scaling parameter in the non-SPS regime. According to the infinite L.E. in the localized region (Eq.(7)) $\gamma_0 \propto \sigma^2_{\ln(t)} E^\eta$ and its variance as $N\sigma^2_\gamma \propto \sigma^2_{\ln(t)}$, it can be seen that the measure of SPS (scaling parameter τ) can be proposed to have a linear relation with the dimensionless variable $\kappa' = \lambda/N_{cr}$.

$$\frac{1}{\tau} \propto E^\eta \propto \frac{\lambda}{N_{cr}} \tag{13}$$

This equation shows a deviation from the SPS value (unity) in the non-SPS regime. The cross-over length N_{cr} plays the role of a length scale like ℓ_s in this system. The above form for the scaling parameter is independent of the system parameters such as disorder strength when it is expressed in terms of τ and κ'. This expression is confirmed by Fig.(6) which shows the inverse scaling parameter versus κ'. It can be seen that for $\kappa' \ll 1$, all data for different disorder strengths, coincide with each other on a single curve. However, since in the second critical point ($\kappa' \approx 1$), the variance of L.E. is energy dependent and also, the scaling form of L.E. (Eq.(7)) is not correct in this point (Fig.(3)), curves with different disorder strengths are separated from each other in the transition point.

In the SPS regime, the scaling of L.E. proposed in Eq.(10) $\gamma_0 \propto \sigma^2_{\ln(t)}$ and its variance form as Eq.(6), show that the scaling parameter τ is independent of system parameters and a constant (Fig.(6)).

Therefore, both the new length scale and the cross-over length that are of the same order in the non-SPS regime, can characterize the scaling properties. Fig.(7) compares these two length scales. It shows the ratio of the new length to the cross-over length in terms of energy. In the Non-SPS regime, they both weakly depend on energy, although, in the transition region, the difference is remarkable.

As a result, the cross-over length is proposed as a physical and meaningful interpretation for the new length scale (ℓ_s). On the other hand, we showed that the statistical distribution of L.E. is different for sizes lower or greater than this length scale.

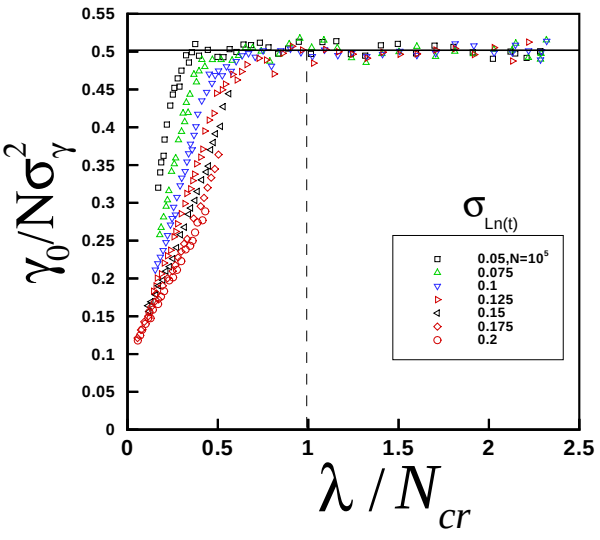

FIG. 6: Inverse scaling parameter $1/\tau = \gamma_0/(N\sigma^2_\gamma)$ versus localization length in the ratio of the cross-over length ($\kappa' = \lambda/N_{cr}$) for different disorder strengths. Energies depending on disorder strength, are scanned from $E = 10^{-7} - 10^0$.

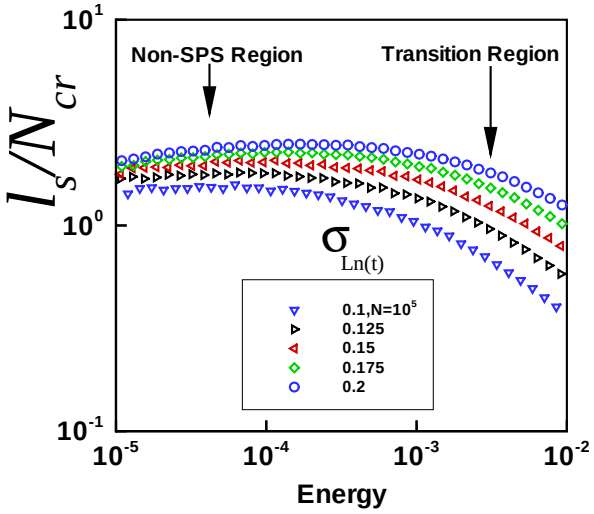

FIG. 7: The ratio of the new length scale (ℓ_s) to the cross-over length in terms of energy for different values of disorder strength.

VI. CONCLUSION

In this paper, we study the scaling theory in the hopping disorder model. The main result of this paper is to show the single parameter scaling (SPS) is violated not only in a region with chiral symmetry, but also in the localized region where there exists a standard symmetry class. The localized region is also divided into two regimes: SPS and non-SPS regimes. We proposed the scaling relations for the Lypunov Exponent in these two regimes. The criterion of the SPS is controlled by a new length scale which is related to the integral of density of states, ℓ_s defined in Ref.[8]. The SPS holds when the localization length λ exceeds the new length ($\lambda \gg \ell_s$). In $\lambda \ll \ell_s$ regime, standard deviation of the Lyapunov Exponent (γ) distribution can be described by two independent scaling parameters: the mean of γ and $\kappa = \lambda/\ell_s$.

We showed that all data related to the variance and mean of the Lyapunov Exponent with different values of disorder strengths, system sizes and also the data extracted from various energy regions, lie on a single curve, when they are expressed in terms of the inverse scaling parameter $1/\tau = \gamma_0/N\sigma^2_\gamma$ and the dimensionless variable κ.

The cross-over length (N_{cr}) which separates the region with chiral symmetry from that of standard symmetry, is proposed as a meaningful physical interpretation for ℓ_s.

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