Statistical properties of a localization-delocalization transition induced by correlated disorder

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The exact probability distributions of the resistance, the conductance and the transmission are calculated for the one-dimensional Anderson model with long-range correlated off-diagonal disorder at $E = 0$. It is proved that despite of the Anderson transition in 3D, the functional form of the resistance (and its related variables) distribution function does not change when there exists a Metal-Insulator transition induced by correlation between disorders. Furthermore, we derive analytically all statistical moments of the resistance, the transmission and the Lyapunov Exponent. The growth rate of the average and typical resistance decreases when the Hurst exponent H tends to its critical value ($H_{cr} = 1/2$) from the insulating regime. In the metallic regime $H \geq 1/2$, the distributions become independent of size. Therefore, the resistance and the transmission fluctuations do not diverge with system size in the thermodynamic limit.

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I. INTRODUCTION

According to the pioneering work of Anderson [1], any amount of disorder localizes all electrons in one-dimensional disordered systems. However, in the case of off-diagonal disorder, due to chiral symmetry, there is peculiar properties such as, divergence of the density of states and the localization length at the band center [2,3,4,5]. Scaling studies of the Anderson model with diagonal disorder and a number of other models of disordered 1D systems, have found that the dimensionless resistance $\rho = R/T$ [6] satisfies $\ln(< \rho >) \propto N$ and $< \ln(1+\rho) > \propto N$ as N the length of the sample goes to infinity [7,8,9]. Purely off-diagonal disorder at $E = 0$, the same as diagonal case for $< \rho >$, shows an exponential growth with the length, but in contradiction with diagonal disorder $< \ln(1+\rho) > \propto \sqrt{N}$ [10,11]. In the case of diagonal disorder, higher moments of the resistance and the transmission can be calculated analytically by using the generalized transfer matrices method [12].

In higher dimensions ($d > 2$), weak disorder does not destroy the metallic regime. Only when the strength of disorder exceeds a critical value, the electrons become localized [1]. This phenomena which is called Anderson transition, does not depend on the microscopic details of the system and is universal. According to the scaling theory of Abrahams, et al [13], there exists a single parameter, conductance g, which determines scaling properties of $g(N)$. Soon it became clear that the conductance g is not a self-averaged quantity. The knowledge of the mean value $< g >$ is therefore not sufficient for complete description of the transport properties. One has to deal with the conductance distribution $G(g)$ [9,14] or equivalently, with all cumulants of the conductance. This is easier in the metallic regime, where $G(g)$ is Gaussian and the conductance fluctuations are universal [15] and independent on the value of the mean conductance and/or the system size. The width of the distribution depends only on the dimension, the physical symmetry of the system and depend on the boundary conditions. In the insulator, large fluctuations of the conductance is characterized by the log-normal distribution of g. Numerical studies [14,16,17] proved the system-size invariance of $G(g)$ at the critical point, which is consistent with the scaling theory of localization. The shape of the distribution is, however, not completely understood.

In addition to the Anderson transition induced by disorder strength, spatially correlation of disorders also can change localization behavior of the system. A short-range spatially correlation of the onsite disorder causes a discrete number of extended states in the random dimer model[18]. This demonstration revived interests in 1D disordered models. Special attention has been recently paid to the presence of a continuum of extended states in 1D around the band center in the long-range correlated disorder model [5,19,20].

Motivated by the distribution evolution of the Anderson transition, in the present paper, we address to answer a question about the conductance (and its related variables) distribution of the long-range correlated off-diagonal disorder along the metallic to insulating transition induced by the correlation between disorders. Our calculations are done analytically at the band center ($E = 0$). It is proved that despite of the Anderson transition in 3D induced by the disorder strength, the functional form of the conductance distribution does not change when a phase transition occurs by means of disorder correlation. It will be also proved that the conductance distribution at the phase transition point and also in the metallic regime, has size invariance, while in 3D Anderson transition, only the critical distribution is size independence.

Furthermore, we derive analytically all statistical moments of the transmission, the resistance and the Lya-

punov Exponent and discuss about their fluctuations. It has been proved that for long-range correlated hopping disorder in the anomalously localized regime $H < 1/2$, $< \rho >$ and also the typical resistance $\tilde{\rho} = \exp[< \ln(1+\rho) >] - 1$ behaves with system size as $e^{N^{1-2H}}$ and $e^{N^{1/2-H}}$ respectively when $N \to \infty$. Here, H refers to the Hurst exponent. The critical Hurst exponent where occurs a localization-delocalization transition, was proved to be $H_{cr} = 1/2$ in this model [5]. The growth rate of the resistance with the length decreases when the Hurst exponent goes to the transition point in $H_{cr} = 1/2$. In the extended regime $H > 1/2$, fluctuations of the transmission and the resistance do not diverge with system size.

This article is organized as the following sections: Section **II** introduces our model and our definition of the Lyapunov Exponent (L.E.). The transmission distribution and its higher moments will be calculated in Section **III**. The conductance distribution and the divergence of its moments are discussed in Section **IV**. Section **V** focuses on the resistance distribution and all its moments. Numerical results on the distribution of the L.E. and analytical results of higher moments at the band center are presented in Section **VI**. Finally, discussions and conclusions are presented in Section **VII**.

II. PURELY HOPPING DISORDER MODEL

We consider electrons in 1D disordered system within a tight binding approximation. The Schroedinger equation by the nearest neighbor assumption becomes

$$\varepsilon_i \psi_i + t_{i,i+1}\psi_{i+1} + t_{i-1,i}\psi_{i-1} = E\psi_i \tag{1}$$

where E is the energy corresponding to the electron wave function. $|\psi_{\mathbf{i}}|^2$ is the probability of finding the electron at site i, ε_i are the site potentials which are considered here to be zero, and $t_{i-1,i} = t_{i,i-1} = t_i$ the hopping terms. Using the transfer matrix method and the above Schroedinger equation, the electron wave functions at the two ends of the system is related to each other by use of total transfer matrix. The total transfer matrix is defined as:($T_{N,0} = \prod_{i=1}^{N} T_{i,i-1}$), where $T_{i,i-1}$ relates two wave functions at neighboring sites. A periodic boundary condition on hopping terms as $t_1 = t_{N+1}$, causes to have unity determinant of total transfer matrix for any configuration of disorder. As proved in Ref.[5], the L.E with the presence of the above condition can be written in the following form.

$$\gamma = \lim_{N \longrightarrow \infty} \frac{1}{N} < |\mathbf{F}| >_{c.a.} \tag{2}$$

where $\mathbf{F} = \ln(|\mathbf{a}|)$ and $\mathbf{a}$ is an arbitrary eigenvalue of the total transfer matrix. $< ... >$ refers to the average of the configurations. In the whole of this paper, our calculations are in the special energy, $E = 0$. In this energy, the total transfer matrix can be easily derived analytically. The function F can be derived as; $\mathbf{F} = \sum_{i=1}^{k} [\ln(\frac{t_{2i-1}}{t_0}) - \ln(\frac{t_{2i}}{t_0})]$ where $\ln(t_0) =< \ln(t_i) >_{c.a.}$. Randomness is imposed on the $\ln t$'s. By taking a Gaussian distribution for $\ln(t)$'s, the above sum has a Gaussian distribution function with zero mean for enough large system size. Therefore, the L.E of such system has a semi-Gaussian distribution whose mean is given by:

$$\gamma(N) = \sqrt{\frac{2}{\pi}} \frac{\sigma_F}{N} \tag{3}$$

where σ_F^2 can be derived as a function of the pair correlation function of $\ln(t)$'s[5].

$$\sigma_F^2 = \frac{2}{\pi}\{Ng(0) + 2\sum_{\ell=1}^{N-1}(N-\ell)(-1)^{\ell} g(\ell)\}$$
$$g(i-j) =< \ln(\frac{t_i}{t_0}) \ln(\frac{t_j}{t_0}) >_{c.a.} \tag{4}$$

This equation is converted to the uncorrelated case with ($g(\ell) = 0$ for $\ell \neq 0$) and ($g(0) = \sigma^2_{\ln(t)}$). In this case, the variance of F function would be $\sigma_F^2 = N\sigma^2_{\ln(t)}$.

III. TRANSMISSION DISTRIBUTION AT $E = 0$

Transmission of a particle through a one-dimensional random potential has become a much studied problem in the theory of disordered systems. It was shown that only the logarithm of the transmission coefficient obeys central limit theorem, whereas averages of T and T^{-1} become unrepresentative of the ensemble for macroscopically large systems [21]. The logarithm of the transmission is proportional to the L.E. and its average scales linearly with the length of the system. Transmission fluctuations are not damped when the sample length goes to infinity. On the other hand, its higher moments are the same order of its mean. Finite-size L.E. which is a self-averaging quantity in the thermodynamic limit, is related to the transmission and conductance as the following form.

$$\gamma(N) = \frac{-1}{2N}\ln(T) = \frac{1}{2N}\ln(1+\frac{1}{g}) \tag{5}$$

where $g(= T/R)$ and T are conductance and transmission coefficients through the system. Since L.E. distribution at the band center has a semi-Gaussian distribution, so by use of the above equation, transmission distribution $\tau(T)$ has a log-normal distribution.

$$\tau(T) = \frac{1}{\sqrt{2\pi\sigma_F^2}} \frac{\exp[-\frac{(\ln T)^2}{8\sigma_F^2}]}{T} \tag{6}$$

The n'th moment of transmission can be easily extracted by using the above distribution.

$$< T^n >= \exp(2n^2\sigma_F^2)\mathrm{erfc}[n\sqrt{2\sigma_F^2}] \tag{7}$$

where erfc(x) is the complementary error function, commonly denoted, is an entire function defined by:

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt \tag{8}$$

where for large x, it goes asymptotically to zero as $\frac{e^{-x^2}}{\sqrt{\pi}x}(1-\frac{1}{2x^2}-...)$, and for small x it tends to unity as $(1-\frac{2e^{-x^2}}{\sqrt{\pi}}(x+\frac{2x^3}{1.3}+...))$.

Depending on the disorder correlation, the variance of (σ_F^2) F can be dependent or independent of the length. In the present paper, we focus on the long-range correlation between disorders which results in a phase transition from the insulating to metallic phase. For long-range correlated disorder, we consider the fluctuations of the $\ln(t)$'s are given by the following [19]:

$$< [\ln(\frac{t_i}{t_0}) - \ln(\frac{t_j}{t_0})]^2 >_{c.a.} = 2\sigma^2_{\ln(t)}|\frac{i-j}{\ell_c}|^{2H} \tag{9}$$

where $\sigma^2_{\ln(t)}$ is kept fixed for all system sizes[22,23] and H is Hurst Exponent which determines the strength of correlation. The correlation length (ℓ_C) is considered to be equal to the system size. i,j are the positions of the bonds along the chain. The pair correlation function arising from Eq.(9), results in the following expressions for variance of F.

$$\lim_{N\longrightarrow\infty}\sigma_F^2(E=0) \propto \begin{cases} \sigma^2_{\ln(t)}N^{1-2H} & H<\frac{1}{2} \\ \sigma^2_{\ln(t)} & H\geq\frac{1}{2} \end{cases} \tag{10}$$

As shown in Ref.[5], an off-diagonal disordered chain with correlation exponent of $H<\frac{1}{2}$ is anomalously localized, while in the case of $H\geq\frac{1}{2}$, the system has a metallic behavior. Based on the above equation, the transmission distribution is independent of size for $H\geq\frac{1}{2}$. Furthermore, the distribution of all statistical variables in the metallic regime of 1D off-diagonal disorder (such as transmission and conductance) are independent of size, while in 3D Anderson model, only the critical distribution is size independence. It needs to mention that in this model in compared with 3D Anderson model, the metallic regime is induced by the long-range correlation between disorders. According to Eq.(10), although the width of the distributions changes through the phase transition, but the whole shape of the distribution functions does not change. The distributions of the conductance and the resistance will be presented in Sections (IV,V).

Higher moments of the transmission in the anomalously localized region, $H<\frac{1}{2}$, can be derived as $N\to\infty$.

$$\lim_{N\longrightarrow\infty} < T^n > \sim \frac{1}{\sqrt{2\pi\sigma_F^2}}\frac{1}{n} \sim \frac{< T >}{n} \;\;;\;\; H<1/2 \tag{11}$$

All moments of the transmission are the same order of its mean. According to the Eq.(10) for $H<\frac{1}{2}$, they go to zero with size as a power law behavior ($N^{H-\frac{1}{2}}$). Furthermore, in the insulating regime, the fractional variance of the transmission (the ratio of the variance to the square of the mean) in the thermodynamic limit diverges to infinity.

$$\lim_{N\to\infty} Var(T)/ < T >^2 \to \infty \tag{12}$$

This behavior of the fractional variance shows that $< T >$ is not a good representative of the statistical ensemble. Therefore the fluctuation of the transmission diverges for large samples.

A perfect transmission through the chain for large chain lengths behaves as $\tau(T=1)\sim N^{H-\frac{1}{2}}$. The scaling behavior of this peak is enough to explain the same behavior for the mean value of the transmission given that other peaks of the distribution decay in a similar or faster way with size of the chain. In the case of uncorrelated disorder which corresponds to $H=0$, the mean value of the transmission has an inverse square root law in terms of the length [24]. This is an interesting behavior since it perfectly coincides with scaling predictions for wide wires of odd number of transversal modes [25].

In the metallic regime with $H\geq\frac{1}{2}$, the variance of F is independent of system size. Therefore, moments of the transmission are independent of two variables; size and correlation exponent. They only depend on the disorder strength. For a weak disorder with condition $\sigma_{\ln(t)} \ll 1/n$, moments of transmission can be expanded in terms of the disorder strength.

$$< T^n >_{N\to\infty} \sim 1 - \sqrt{\frac{8}{\pi}}n\sigma_{\ln(t)} \;\;;\;\; H\geq\frac{1}{2} \tag{13}$$

In the metallic regime, also it can be simply shown that the fractional variance goes to a constant in the thermodynamic limit. Therefore, the transmission can be a representative variable for describing statistical properties of the 1D metallic system.

IV. CONDUCTANCE DISTRIBUTION

The distribution of the conductance can be obtained by using its relation with L.E. in Eq.[5].

$$G(g) = \frac{1}{\sqrt{2\pi\sigma_F^2}}\frac{\exp[-\frac{\ln(1+\frac{1}{g})^2}{8\sigma_F^2}]}{g(1+g)} \tag{14}$$

This distribution converges to a power law form $1/(g^2\sigma_F)$, as it should be for large g, but for small g the distribution behaves log-normal form as: $G(g) \propto \exp[-\frac{\ln(g)^2}{8\sigma_F^2}]/(g\sigma_F)$. In the metallic regime, where $H \geq \frac{1}{2}$, variance of F will be independent of the system size. So, the conductance distribution function will be invariant as $N \to \infty$. In the insulating regime, $H < \frac{1}{2}$, Eq.(10) shows that σ_F behaves with size as $N^{1/2-H}$ in the thermodynamic limit. Therefore, in this regime, the conductance distribution for large g, goes off more faster than the metallic one. It tends to zero as $N^{H-1/2}/g^2$. It means that in the insulator regime, large conductance occurrence happen rarely with a low probability. For $g \to 0$, the conductance distribution goes to zero $G(g) \to 0$. However, two limits of $g \to 0$ and $N \to \infty$ do not commute with each other.

Because the conductance distribution decays very slow $G(g) \propto g^{-2}$ as $g \to \infty$, all moments of g diverge. Large conductance can cause to diverge the mean of g in the averaging process, although occurrence probability of large conductance is very low. Therefore, for omitting the effect of large conductance in the mean process, it is better to mean the resistance $\rho = 1/g$ instead of the conductance.

V. RESISTANCE DISTRIBUTION

The resistance distribution at the band center can be also derived by using the definition in Eq.(5).

$$R(\rho) = \frac{1}{\sqrt{2\pi\sigma_F^2}} \frac{\exp[-\frac{[\ln(1+\rho)]^2}{8\sigma_F^2}]}{1+\rho} \tag{15}$$

This distribution goes to $(1-\rho)/\sqrt{2\pi\sigma_F^2}$ as it should be for small resistance ρ. So, $R(\rho)$ is an ascending function of ρ when $\rho \to 0$. But for large ρ, this distribution converges to the log-normal form. The n'th moment of the resistance at $E = 0$ can be derived as the following summation.

$$< \rho^n >= \sum_{\ell=0}^{n} \frac{(-1)^\ell n!}{(n-\ell)!\ell!} e^{2(n-\ell)^2\sigma_F^2} \mathrm{erfc}[-\sqrt{2(n-\ell)^2\sigma_F^2}] \tag{16}$$

In the anomalously localized regime ($H < 1/2$), where σ_F^2 goes to infinity in the thermodynamic limit, the above equation can be summarized as the following form.

$$< \rho^n >= (-1)^n + 2\sum_{\ell=0}^{n-1} \frac{(-1)^\ell n!}{(n-\ell)!\ell!} e^{2(n-\ell)^2\sigma_F^2} \tag{17}$$

As an immediate consequence of the above general formula, the average resistance grows with system size as:

$$< \rho > \propto e^{2\sigma^2_{\ln(t)} N^{1-2H}} \tag{18}$$

Since $\frac{<\rho^n>}{<\rho>^n} \sim e^{2n(n-1)\sigma_F^2}$, the fractional variance (n=2) of the resistance diverges as $N \to \infty$ for $H < 1/2$. Furthermore, all moments of the resistance distribution also diverges. According to the results of Anderson *et al* [9], instead of the resistance whose moments diverge, the appropriate quantity is a self-averaged variable $\ln(1+\rho)(= 2N\gamma)$ which its fractional variance goes to zero as $N \to \infty$. However, the fractional variance of the new variable in the hopping disorder model at $E = 0$ tends to a non-zero constant $(\pi/2 - 1)$. This constant can be derived by using a relation between the second moment and mean of the L.E. as $< \gamma^2 >= \pi/2 < \gamma >^2$. Furthermore, the average $\ln(1+\rho)$ has already calculated through the Eqs.(3,10) as:

$$< \ln(1+\rho) > \propto \begin{cases} \sigma_{\ln(t)} & H \geq \frac{1}{2} \\ \sigma_{\ln(t)} N^{1/2-H} & H < \frac{1}{2} \end{cases} \tag{19}$$

The typical resistance is introduced as a candidate variable for experimental resistance. Using the above equation, in the thermodynamic limit, this measurable quantity behaves as $\tilde{\rho} \propto e^{N^{1/2-H}}$ with the system size for $H < 1/2$. In the special case of $H = 0$ which corresponds to the uncorrelated disorder, in good agreement with the results of Refs.[10,11], the average and typical resistances are as e^N and $e^{\sqrt{N}}$. It is clear that the growth rate of the average and typical resistance decreases when the Hurst exponent tends to its critical value $H_{cr} = 1/2$.

In the metallic regime $H > 1/2$, the resistance only grows with the disorder strength. Also, in this regime, the fractional variance of the resistance is independent of size and also the Hurst exponent. This expression means that fluctuations of the resistance and the transmission do not increase with size when $N \to \infty$. Therefore, despite of the insulating regime, they can be considered as representative variables for the statistical properties of transport.

VI. LYAPUNOV EXPONENT DISTRIBUTION

With care of our discussions about the transmission, the conductance and the resistance properties, the appropriate variable for describing localization properties is the Lyapunov Exponent. It can be shown that the L.E. distribution function and all its higher moments converge for large system sizes.

Fig.(1) shows numerical calculations of the L.E. distribution for different Hurst Exponents [5]. All distribution curves have been softened by the Kernel smoothing method [26] without changing any statistical characteristic of distributions. It can be seen that the functional shape of the L.E. distribution does not change when a phase transition occurs by the correlated disorder. The pair correlation function arising from Eq.(9), results in

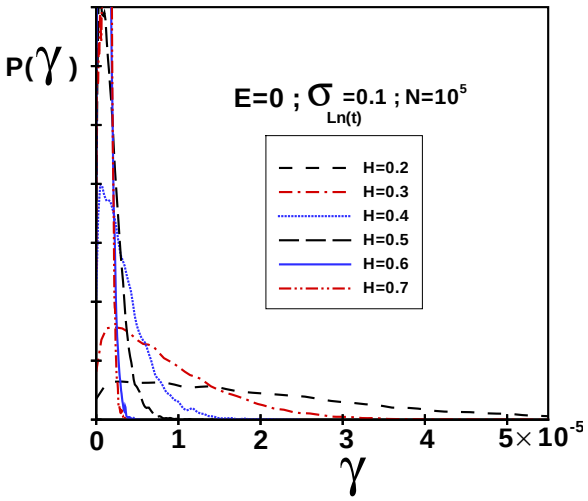

FIG. 1: Lyapunov Exponent Distribution Function for different Hurst Exponents at $E = 0$. The disorder strength is considered to be 0.1. The sample size is 10^5.

the following expression for the L.E variance as $N \to \infty$.

$$\sigma_\gamma^2 \propto \begin{cases} \frac{\sigma_{ln(t)}^2}{N^2} & H \geq \frac{1}{2} \\ \frac{\sigma_{ln(t)}^2}{N^{1+2H}} & H < \frac{1}{2} \end{cases} \qquad (20)$$

This equation shows that correlated disorder causes the variance of the L.E. distribution function to converge faster than in the uncorrelated case (H=0). It can be seen that in good agreement with Anderson *et al.* [9], the variance of the L.E. in the uncorrelated disorder scales according to the law of large numbers as $1/N$.

With care of the semi-Gaussian distribution function for the L.E, it can be simply proved that the n'th moment of the L.E is proportional to the n'th power of its mean as $< (\gamma - < \gamma >)^n > \propto < \gamma >^n$. So, Eq.(20) can be generalized to higher moments of the L.E. as the following scaling law.

$$< (\gamma - < \gamma >)^n > \propto \begin{cases} (\frac{\sigma_{\ln(t)}}{N})^n & H \geq 1/2 \\ (\frac{\sigma_{\ln(t)}}{N^{1/2+H}})^n & H < 1/2 \end{cases} \qquad (21)$$

So, all moments of the L.E. in the metallic regime ($H > 1/2$) are independent of the Hurst Exponent. Fig.(1) shows that the variance of L.E. does not change for $H > 1/2$. Of course, at the phase transition point ($H = 1/2$), the width of the distribution will tend to $H > 1/2$ values in the thermodynamic limit.

VII. CONCLUSION AND DISCUSSION

The distribution functions of the resistance, the conductance and the transmission was analytically derived for 1D off-diagonal Anderson model with long-range correlated disorder at the band center (E=0). Despite of the Metal-Insulator transition induced by the disorder strength in 3D, the phase transition induced by the correlated disorder in 1D does not cause to change the shape of the distribution function of the conductance (and all related variables). In the metallic regime, the distribution functions does not depend on the system size, while in 3D Anderson model only the critical distribution is size independence.

We derived analytically all statistical moments of the resistance, the transmission and the Lyapunov Exponent. In the anomalously localized regime $H < 1/2$, by means of these moments, it was shown that the fluctuations of the transmission, the conductance and the resistance diverge in the thermodynamic limit. Convergence of the Lyapunov Exponent will happen not only with the system size but with the correlation exponent. The growth rate of the average and typical resistances decrease when the correlation exponent closes to its transition point from insulating to metallic regime. In the metallic regime $H > 1/2$, the resistance and the transmission fluctuations do not diverge with the system size.

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